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ON THE SOPHISTICATION OF FINANCIAL INVESTORS AND THE INFORMATION REVEALED BY PRICES

Andrés Carvajal*[1,2] and Hang Zhou[3]

[1]University of California, Davis, USA

[2]EPGE-FGV, Brazil

[3] School of Finance, Shanghai University of Finance and Economics, China

Abstract. Does increasing the traders' understanding of the financial markets lead to their more orderly functioning? We provided an answer by studying a simple exchange economy with asymmetric information. Each trader was either a fundamentalist, who knew the probability distribution of random shocks, or a speculator trying to infer that distribution from asset prices. Starting with the unsophisticated beliefs that asset prices transfmitted no information, the speculators learned the mapping from asset prices to probabilities through adaptive observation. Our results characterized the necessary conditions for convergence to rational expectations equilibrium.

1. **Introduction.** Rational expectations equilibrium (REE, henceforth) has been an essential idea in economic theory since seminal work by [9]. [11] models REE as self-fulfilling beliefs, where agents maximize their utility based on their beliefs, and the market-clearing outcome confirms these beliefs. Interestingly, [11] proves the generic existence of an REE where all the agents can figure out all the private information in the economy once they observe the prices.

However, the epistemic demands of REE are significant: the mapping from states to prices (and its inverse) is common knowledge among all the agents, while the model is silent on how the agents would learn this mapping. To address these limitations, we incorporate adaptive learning into a general equilibrium setting and define an iterative process where the agents' mappings evolve as they become "more sophisticated." Our result provides conditions under which the mappings from states to prices converge to the REE.

The mathematics of our model resemble game theoretical models of level-k reasoning.[1] In those models, level-k reasoning is an alternative to the Nash equilibrium that describes how strategic sophistication determines players' strategies. Here, despite the resemblance, we do not interpret the evolution of beliefs as an issue of reasoning. Instead, we imagine the agents as learning through the observation of

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*Corresponding author: Andrés Carvajal.

[1] See [4] for a thorough review on level-k thinking models and its supporting experimental evidence. [15] incorporates level-k reasoning into a non-competitive financial market in the spirit of [8] and explores whether it converges to REE, which is also the Nash equilibrium.

reality: after trading, the less informed agents discover the information they lack and use this observation to try to figure out the structure of the mapping that determines prices as a function of the available information. If these agents were fully rational, they would realize that just by learning about such a mapping, the relation between the two variables may change; indeed, REE is the formalization of the idea that the equilibrium mapping is one that is consistent with itself, in the sense that it is the mapping that results when all agents are using it. On the other hand, our agents follow an adaptive learning process: once they learn a mapping, they start using it, and it takes them some time to observe the resulting relation between prices and information, namely, the next mapping they will use.

Our paper is also related to [2], [6], and [7], which study the stability of the REE. [2] surveys the literature on convergence to the REE. They model the learning process as agents estimating a parameter with some likelihood functions. [6] shows that convergence to REE is possible even with heterogeneous initial belief. Our paper is different in two respects. First, we focus on learning the REE mapping instead of a single parameter. Second, our agents are not Bayesian and learn adaptively.

A more related paper is [7]. This study examines an adaptive learning process in which agents are uncertain about the stability of the equilibrium and extrapolate from past trends. The paper demonstrates that near-equilibrium dynamics play a critical role in the convergence to REE. Convergence is assured only if agents do not excessively extrapolate past trends. In comparison, our findings suggest that convergence (or divergence) occurs when speculators are less (or more) responsive than fundamentalists. Both papers highlight the potential link between divergence and the excessive amount of adaptive behavior.

This paper is also related to [10], which studies an economy where agents update their private information based on the observed REE prices and allocations. They, too, show convergence to the symmetric information REE, but their learning process is different. At each round of trade, a REE is observed by all the agents, and they use this information to refine their state-space partitions for the next round of trade.[2] This means that the agents always infer correct information but become more refined across rounds. In our case, no round of trade needs to result in an REE, and the agents here may infer wrong information from the prices they observe. The emphasis in [10] is on the convergence of their sequence of REE to a symmetric REE. For us, the focus is on whether a sequence of non-REE price functions converges to an REE.

2. **A simple radner economy.** While our argument can be made in a more general setting, for simplicity we consider a minimal, two-period exchange economy with uncertainty, where the state space for the future period is $\Sigma = \{1, 2\}$. There is only one commodity in the economy, and consumption takes place only in the second period.

There are two types of agent in the market. *Fundamentalists* know that the probability that state of the world $\sigma = 1$ will realize in the future is $\pi \in \Delta \subseteq (0, 1)$. *Speculators* don't know this, and initially only understand that π is a realization of random variable Π, whose support is Δ. We will use the super-index $a \in \{F, S\}$ to denote the agents' types.[3]

[2] They also refine their understanding of their endowments.

[3] We may also name the agents as the informed and the uninformed traders. However, the uninformed traders do not perform Bayesian learning on equilibrium prices. We name them as Fundamentalists and Speculators to distinguish from the asymmetric information models.

Besides their information, the two types may differ in their future wealth. In state σ, agents of type a will be endowed with a wealth ω^a_σ. We assume that there is a continuum of agents of each type, with respective masses μ^F and μ^S.

In the present, the agents trade the elementary securities corresponding to the two states of nature. They have expected-utility preferences with type-dependent Bernoulli utility function $u^a : \mathbb{R} \to \mathbb{R}$. Denoting the holdings by agents of type a of the security that pays in state σ as y^a_σ, the ex-ante utility of a fundamentalist is

$$\pi \cdot u^F(\omega^F_1 + y^F_1) + (1-\pi) \cdot u^F(\omega^F_2 + y^F_2); \tag{1}$$

for a speculator, if she receives or discerns information $\mathcal{I} \subseteq \Delta$, it is

$$\mathrm{E}\left[\Pi \cdot u^S(\omega^S_1 + y^S_1) + (1-\Pi) \cdot u^S(\omega^S_2 + y^S_2) \mid \mathcal{I}\right]. \tag{2}$$

What information the speculators use will depend, of course, on their understanding of the market.

We normalize the price of the security for $\sigma = 2$ to unity, and denote by q the price of the security for $\sigma = 1$. When an agent of type a chooses her portfolio of the two securities, the only constraint she faces is that

$$q \cdot y^a_1 + y^a_2 = 0. \tag{3}$$

Note that one can use Eq. (3) to solve for the holdings of the second security and then rewrite Eqs. (1) and (2) in terms of the first security only. Using this simplification, we can write the optimal demands for the first security as $Y^F(q, \pi)$ for the fundamentalists, and as $Y^S(q, \mathcal{I})$ for the speculators.[4]

Market clearing requires that the aggregate of the two types' demands vanish:

$$\mu^F \cdot Y^F(q, \pi) + \mu^S \cdot Y^S(q, \mathcal{I}) = 0. \tag{4}$$

3. **Rational expectations.** An REE is a function $\bar{Q} : \Delta \to \mathbb{R}$ such that, for all $\pi \in \Delta$, Eq. (4) is satisfied for $q = \bar{Q}(\pi)$ and

$$\mathcal{I} = \bar{Q}^{-1}(q). \tag{5}$$

The equilibrium is *fully revealing* if it is injective, namely, if

$$\bar{Q}^{-1}(\bar{Q}(\pi)) = \{\pi\} \tag{6}$$

for all π.

Equation (5) requires that speculators discern all the information transmitted by prices *correctly at equilibrium*. Eq. (6) further requires that prices transmit *all* of the information available to the fundamentalists. It has been known, since [11], that mild conditions on preferences guarantee the existence of fully-revealing REE, generically on the agents' wealth levels.

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[4] Formally,

$$Y^F(q, \pi) = \arg\max_y \left\{\pi \cdot u^F(\omega^F_1 + y) + (1-\pi) \cdot u^F(\omega^F_2 - q \cdot y)\right\}$$

and

$$Y^S(q, \mathcal{I}) = \arg\max_y \left\{\mathrm{E}\left[\Pi \cdot u^S(\omega^S_1 + y) + (1-\Pi) \cdot u^S(\omega^S_2 - q \cdot y) \mid \mathcal{I}\right]\right\}.$$

4. **Adaptive learning by the speculators.** The concept of REE assumes implicitly that the speculators know the equilibrium and use it to infer the information they lack. The epistemic requirements it imposes[5] are not unlike those of Nash equilibrium in game theory.[6]. In game theory, an alternative approach is provided by the model of level-k reasoning: one starts by stipulating what the most naïve behavior of a player is, and proceeds inductively to define higher levels of sophistication as the reasoning of a player who thinks that everybody else in the game is one level below. We adapt this mathematical apparatus to the current setting by assuming that the speculators need to learn the mapping from fundamentals to prices through observation, starting from the behavior of a trader who understands this mapping "the least."[7]

4.1. **Clueless speculators.** We assume that the speculators learn the relation between fundamentals and prices through trading experience. A completely inexperienced speculator would fail to realize that the price depends on the information received by the fundamentalists. We say these least verse speculators have *level-*0 *understanding*, and define the corresponding demand of the speculator as $Y_0^S(q) = Y^S(q, \Delta)$. This clueless trader is one who uses unconditional expectations on her portfolio problem (Eq. (2) with $\mathcal{I} = \Delta$, the full support of Π) regardless of the price q she faces in the market.

Assuming that it exists, we define the market-clearing pricing function $Q_0 : \Delta \to \mathbb{R}$ by the solution of Eq. (4) with level-0 understanding by the speculators; explicitly

$$\mu^F \cdot Y^F(Q_0(\pi), \pi) + \mu^S \cdot Y_0^S(Q_0(\pi)) = 0,$$

for all π.

4.2. **Learning.** For any natural number k, let the function $Q_{k-1} : \Delta \to \mathbb{R}$ be given. We say that a speculator has *level-k understanding* if she understands the dependence of prices on the information of the fundamentalists through price function Q_{k-1}. At prices q, this speculator uses information $\mathcal{I} = Q_{k-1}^{-1}(q)$ in her choice of an optimal portfolio. Her optimal demand for the first security can thus be denoted as

$$Y_k^S(q) = Y^S(q, Q_{k-1}^{-1}(q)).$$

When the speculators display level-k understanding, a new pricing function arises: $Q_k : \Delta \to \mathbb{R}$, defined by

$$\mu^F \cdot Y^F(Q_k(\pi), \pi) + \mu^S \cdot Y_k^S(Q_k(\pi)) = 0,$$

assuming that such a function exists.

Importantly, the speculators do *not* realize that their use of function Q_{k-1} changes the equilibrium prices at each value of π — namely, that it induces the new mapping Q_k.[8]

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[5] See [5] and [1].

[6] A Nash equilibrium "requires" that all players have common knowledge of rationality and they believe that others will also play the equilibrium strategy with full probability. Therefore, the strategic uncertainty is resolved and players' private types or preferences are revealed.

[7] We will continue to use the language of the level-k model, but insist that we maintain the Walrasian interpretation that the traders in a market like the one we are modeling do not engage in higher-order reasoning about the beliefs or rationality of others.

[8] The existence and uniqueness of the mapping Q_k depends on the monotonicity of the demand function Y_k^F and Y_k^S. To be sure, though, there is no general guarantee of this monotonicity, even

4.3. **Rational expectations again.** Let $\mathcal{Q}$ be the space of functions $Q : \Delta \to \mathbb{R}$. Note that, starting from Q_0, the definition of level-k understanding recursively constructs a sequence in $\mathcal{Q}$. Let us denote by $\mathcal{R}$ the mapping that defined the recursion $Q_{k-1} \mapsto \mathcal{R}(Q_{k-1}) = Q_k$. By construction, any REE is a fixed point of $\mathcal{R}$. The question that will occupy us is whether there exist conditions that ensure that sequence $\langle Q_k \rangle_{k\in\mathbb{N}}$ converges to $\bar{Q}$.

5. **Convergence to rational expectations.** Assume that both types of agent have $\mathcal{C}^2$, strictly increasing and strictly concave Bernoulli indexes that yield demand functions $Y^a(q, \pi)$ that are decreasing in q and increasing in π.[9]

Critically, let us assume that the learning process allows the speculators to discern beliefs uniquely, in the sense that all the price functions Q_k are bijective. The operator $Q \mapsto \mathcal{R}(Q)$ is implicitly defined by

$$\mu^F \cdot Y^F(\mathcal{R}(Q)(\pi), \pi) + \mu^S \cdot Y^S(\mathcal{R}(Q)(\pi), Q^{-1}(\mathcal{R}(Q)(\pi))) = 0,$$

for all $\pi \in \Delta$. If we denote by $\mathcal{T}$ the mapping that associates the inverse of Q to the inverse of $\mathcal{R}(Q)$, we can replace the recursion of price functions by the recursion of belief functions $\hat{\Pi} \mapsto \mathcal{T}(\hat{\Pi})$ defined by

$$\mu^F \cdot Y^F(q, \mathcal{T}(\hat{\Pi})(q)) + \mu^S \cdot Y^S(q, \hat{\Pi}(q)) = 0 \tag{7}$$

for all $q \in Q(\Delta)$.

More explicitly, Eq. (7) tells us that level-k learning is the implicit recursion

$$\mu^F \cdot Y^F(q, \hat{\Pi}_k(q)) + \mu^S \cdot Y^S(q, \hat{\Pi}_{k-1}(q)) = 0, \tag{8}$$

starting from the least versed beliefs $q \mapsto \hat{\Pi}_0(q) = \mathrm{E}(\Pi)$.

Lemma 5.1. *When the realized π happens to be $\mathrm{E}(\Pi)$, the price that clears the markets is the same for all levels of understanding and equals the price that clears them under rational expectations. That is, $Q_k(\mathrm{E}(\Pi)) = \bar{Q}(\mathrm{E}(\Pi))$ for all k.*

Proof. The beliefs of level-0 speculators are the constant mapping $q \mapsto \hat{\Pi}_0(q) = \mathrm{E}(\Pi)$. Suppose now π equals $\mathrm{E}(\Pi)$. The market clearing condition is

$$\mu^F \cdot Y^F(q, \mathrm{E}(\Pi)) = -\mu^S \cdot Y_0^S(q),$$

while the market clearing condition under fully-reveling rational expectations is

$$\mu^F \cdot Y^F(q, \mathrm{E}(\Pi)) = -\mu^S \cdot Y^S(q, \mathrm{E}(\Pi)).$$

Since the latter has a unique solution $q = \bar{Q}(\mathrm{E}(\Pi))$, it follows that $Q_0(\mathrm{E}(\Pi)) = \bar{Q}(\mathrm{E}(\Pi))$.

Now, suppose that $Q_{k-1}(\mathrm{E}(\Pi)) = \bar{Q}(\mathrm{E}(\Pi))$ for some natural number k. Again, when $\pi = \mathrm{E}(\Pi)$ the market clearing condition requires that

$$0 = \mu^F \cdot Y^F(q, \mathrm{E}(\Pi)) + \mu^S \cdot Y_k^S(q) = \mu^F \cdot Y^F(q, \mathrm{E}(\Pi)) + \mu^S \cdot Y^S(q, Q_{k-1}^{-1}(q)).$$

Under the assumption that $Q_{k-1}^{-1}(\bar{Q}(\mathrm{E}(\Pi))) = \mathrm{E}(\Pi)$, the only solution has $q = \bar{Q}(\mathrm{E}(\Pi))$, so $Q_k(\mathrm{E}(\Pi)) = \bar{Q}(\mathrm{E}(\Pi))$.

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for simple utility function such as the CRRA utility, due to potentially complicated wealth effects. For more details, see the comprehensive survey [14].

[9] For example, an agent with $\omega_1^a = \omega_2^a = \omega^a$ and logarithmic utility has, by direct computation,

$$Y^F(q, \pi) = \left(\pi \cdot \frac{q+1}{q} - 1\right) \omega^F.$$

The lemma, hence, follows by mathematical induction. □

An immediate implication is the following corollary:

Corollary 5.2. *Let $\bar{q}$ be the price that clears the markets under rational expectations when the realized π happens to be $\mathrm{E}(\Pi)$, namely, $\bar{q} = \bar{Q}(\mathrm{E}(\Pi))$. For all levels of understanding, when the speculators observe $\bar{q}$, they believe that the realized π is indeed $\mathrm{E}(\Pi)$. That is, $\hat{\Pi}_k(\bar{q}) = \mathrm{E}(\Pi)$ for all k.*

We can now state and prove a convergence result. The proof relies on the previous corollary in that it allows us to prove convergence of the sequence of belief functions by proving the convergence of their derivative functions. The key assumption of the theorem is that the response of the aggregate demand of the speculators to an increase in the probability of state cannot dominate the response of the fundamentalits' demand.

Theorem 5.3. *If*

$$\frac{\mu^F \frac{\partial Y^F}{\partial q}(q,\pi) + \mu^S \frac{\partial Y^S}{\partial q}(q,\{\pi\})}{\mu^F \frac{\partial Y^F}{\partial \pi}(q,\pi)}$$

is bounded and

$$\sup_{\pi,q}\left\{\frac{\mu^S}{\mu^F}\cdot\frac{\partial Y^S}{\partial \pi}(q,\pi)\cdot\left[\frac{\partial Y^F}{\partial \pi}(q,\pi)\right]^{-1}\right\} < 1,$$

then the sequence of level-k price functions $\langle Q_k\rangle_{k=1}^{\infty}$ converges uniformly to the REE.

Proof. It suffices to show that the sequence $\langle\hat{\Pi}_k\rangle_{k=1}^{\infty}$ converges uniformly to $\bar{\Pi} = \bar{Q}^{-1}$. For this, given the corollary above, it suffices that we argue that the sequence $\langle\hat{\Pi}'_k\rangle_{k=1}^{\infty}$ converges uniformly to $\bar{\Pi}'$.[10]

Define the functions

$$A_k(q) = -\frac{\mu^F \frac{\partial Y^F}{\partial q}(q,\hat{\Pi}_k(q)) + \mu^S \frac{\partial Y^S}{\partial q}(q,\hat{\Pi}_{k-1}(q))}{\mu^F \frac{\partial Y^F}{\partial \pi}(q,\hat{\Pi}_k(q))}$$

and

$$B_k(q) = \frac{\mu^S}{\mu^F}\frac{\frac{\partial Y^S}{\partial \pi}(q,\hat{\Pi}_{k-1}(q))}{\frac{\partial Y^F}{\partial \pi}(q,\hat{\Pi}_k(q))}$$

both of which take only positive values. Differentiating Eq. (8) implicitly, we have that

$$\hat{\Pi}'_k(q) = A_k(q) - B_k(q)\hat{\Pi}'_{k-1}(q). \tag{9}$$

By recursive substitution, that is,

$$\hat{\Pi}'_k(q) = \sum_{j=1}^{k}\left[(-1)^{k-j}A_j(q)\prod_{\ell=j+1}^{k}B_\ell(q)\right] + (-1)^k\prod_{j=1}^{k}B_j(q)\hat{\Pi}'_0(q).$$

Since each A_k is bounded and each B_k is bounded above strictly below 1, the first of these two summands converges uniformly: see Theorem 7.10 in [12]. By the same assumption on each B_k, the second term converges uniformly to the function constant at 0. □

[10] See Theorem 7.17 in [12].

5.1. **Divergence.** Importantly, the proof of the theorem shows that:

Corollary 5.4. *$\langle \hat{\Pi}'_k(q) \rangle_{k=1}^{\infty}$ fails to converge at prices q for which*

$$\frac{\partial Y^S}{\partial \pi}(q, \bar{Q}^{-1}(Q)) \cdot \left[\frac{\partial Y^F}{\partial \pi}(q, \bar{Q}^{-1}(Q)) \right]^{-1} \geq \frac{\mu^F}{\mu^S}. \tag{10}$$

Since the righthand side of this expression can be set arbitrarily close to zero, the argument above implies that point-wise convergence to REE fails for an open subset of prices.

In order to develop intuition for this claim, we can rewrite Eq. (10) and note that convergence fails at any π such that

$$\mu^S \frac{\partial Y^S}{\partial \pi}(\bar{Q}(\pi), \pi) \geq \mu^F \frac{\partial Y^F}{\partial \pi}(\bar{Q}(\pi), \pi).$$

The left-hand side of this expression is the aggregate response of the speculators' demand to an increase in the probability π. The requirement for convergence is that this response always be smaller than that of the fundamentalists' aggregate demand. Whether or not this is the case depends, of course, on the fundamentals of the economy.

5.2. **Wealth evolution.** [3] and [13] have argued that agents who hold wrong beliefs about future risks accumulate dynamic losses that in the long run lead them to bankruptcy. This observation opens a new mechanism through which convergence to REE may occur. When the speculators hold incorrect beliefs on the probability, they sustain losses that dynamically reduce their wealth and, perhaps, aggregate the response of their asset demand to changes in π.

While our model is not genuinely dynamic, we can give a first assessment of the interaction between level-k learning and market losses by introducing to the model the wealth dynamics that would result from the losses that, on average, a level-k speculator sustains as a consequence of her use of her beliefs, as opposed to the correct beliefs.

At prices q, a speculator who believes that the probability of state 1 is $\hat{\pi}$ deviates from her correct optimal demand by $Y^s(q, \hat{\pi}) - Y^s(q, \pi)$. In expectation, their difference in beliefs causes for the speculators an "erroneous" profit of

$$\mathrm{E}\{(\Pi - \hat{\pi}) \cdot [Y^s(q, \hat{\pi}) - Y^s(q, \Pi)]\}$$

in state $\sigma = 1$, and of

$$\mathrm{E}\left\{ (\hat{\pi} - \Pi) \cdot \frac{Y^s(q, \Pi) - Y^s(q, \hat{\pi})}{q} \right\}$$

in state $\sigma = 2$. Since Y^s is increasing in π, these two numbers are negative, and speculators are expected to become poorer in *both* states of the world if they are not extracting the correct probability from the prices they observe.

Now, suppose that the speculators with level-k understanding enter the market with wealth $w_k^S = (w_{k,1}^S, w_{k,2}^S)$. During the process of learning the more informed belief function $\hat{\Pi}_{k+1}$, they sustain the losses

$$\ell_{k,1}^S = -\mathrm{E}\left\{ [\Pi - \hat{\Pi}_k(Q_k(\Pi))] \cdot [Y^s(Q_k(\Pi), \hat{\Pi}_k(Q_k(\Pi))) - Y^s(Q_k(\Pi), \Pi)] \right\}$$

and

$$\ell_{k,2}^S = -\mathrm{E}\left\{ [\hat{\Pi}_k(Q_k(\Pi)) - \Pi] \cdot \frac{Y^s(Q_k(\Pi), \Pi) - Y^s(Q_k(\Pi), \hat{\Pi}_k(Q_k(\Pi)))}{Q_k(\Pi)} \right\},$$

so that their income when they have level-$(k+1)$ understanding is $w^S_{k+1} = w^S_k - (\ell^S_{k,1}, \ell^S_{k,2})$.[11] At the same time, the fundamentalists' income increases by μ^S/μ^F times the losses of the speculators.

An open question is what this income reallocation does to the term

$$\sup_{\pi,q} \left\{ \frac{\mu^S}{\mu^F} \cdot \frac{\partial Y^S}{\partial \pi}(q, \pi; w^s) \cdot \left[\frac{\partial Y^F}{\partial \pi}(q, \pi; , w^F) \right]^{-1} \right\},$$

whose dependence on the traders' incomes we now make explicit. Unfortunately, the comparative statics

$$\frac{\partial^2 Y^a}{\partial w^a_1 \, \partial \pi} \text{ and } \frac{\partial^2 Y^a}{\partial w^a_1 \, \partial \pi}$$

are in general unclear. If these two terms are negative for both types of agents, the dynamics of wealth whereby at every level of learning, and the speculators sustain losses and the fundamentals experience gains that makes term

$$\frac{\mu^S}{\mu^F} \cdot \frac{\partial Y^S}{\partial \pi}(q, \pi; w^s) \cdot \left[\frac{\partial Y^F}{\partial \pi}(q, \pi; w^F) \right]^{-1}$$

decrease as k increases, which favors convergence.[12]

6. **Concluding remarks.** Our motivation for this paper is that, while appealing, the concept of rational expectations imposes demanding presumptions from an epistemic point of view. An agent who only knows about their preferences and does not make considerations about the rationality of others and the macroeconomic feasibility of the market situation would hardly satisfy those epistemic requirements.

We assume, in contrast, that uninformed agents only learn, through observation, in "reduced form." Our speculators don't know if the other traders in the market are rational, or if the other agents think that the other agents are rational, and so on. They don't wonder if the other agents will supply the portfolio they find optimal at some prices at those same prices. Instead, they appeal to their recollection of what the information they lack turned out to be when they traded at those same prices in the past. What they fail to realize is that by using that information, they change the mapping from the relevant information to the market-clearing prices, so they need to revise their knowledge.

Our results provide conditions under which a sequence of non-REE price functions converges to REE. In essence, the conditions require that the mistakes made by the uninformed traders do not overpower the prices' ability to clear the markets. Notably, the condition is not only sufficient, but, in a sense, it also is necessary.

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[11] Start the income sequence with $w^S_0 = \omega^S$.

[12] For example, if both types of individuals have CRRA preferences, so that their marginal utilities of income are of homogeneous of degree $-\rho^a < 0$, then

$$\frac{\partial Y^a}{\partial w^a_1}(q, \pi; w^a) = - \left[1 + q^{\frac{\rho^a - 1}{\rho^a}} \cdot \left(\frac{\pi}{1-\pi} \right)^{\frac{1}{\rho^a}} \right]^{-1}$$

and

$$\frac{\partial Y^a}{\partial w^a_2}(q, \pi; w^a) = q^{-\frac{1}{\rho^a}} \left[\left(\frac{1-\pi}{\pi} \right)^{\frac{1}{\rho^a}} + q^{\frac{\rho^a - 1}{\rho^a}} \right]^{-1}.$$

With both terms being increasing in π, the dynamics of income reallocation from speculators to fundamentalists works in favor of convergence.

For CARA preferences, on the other hand, the second derivatives are zero, so the proposed effect is absent in that case.

When the condition is violated, the orderly functioning of markets is compromised, and the sequence of functions can diverge for some values of the fundamental. When that is the case, convergence may be restored through a different mechanism: the losses sustained by the speculators arising from their wrong inference of the fundamentals.

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