

IDIOSYNCRATIC RISK AND THE EQUITY PREMIUM*

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Abstract

This paper aims to further our understanding of the effect of idiosyncratic risk on the equity premium. We consider different classes of preferences and different co-variations between the idiosyncratic shocks' variance and the economy's aggregate income. We offer a complete characterization of the effect for short-lived assets relying on the cross-moments of different utility function derivatives and the economy's aggregate income. We also study the effects of higher-order moments of the distribution of idiosyncratic risk.

Our comparative statics results present a series of corrections to the theoretical equity premium using a parameterization of the moments of the distribution of idiosyncratic risk. Our approach can be extended and applied in other contexts, but we recognize that no correction corresponds exactly to the equity premium except under extra assumptions. As a test of the robustness of our corrections, we compare them to the exact premium in a simplified setting where the latter can be explicitly computed. The results suggest that the approximation errors implicit in our corrections are at least of second order.

A complete characterization is elusive for long-lived assets, but we present sufficient conditions for reversing the effect on short-lived assets.

Keywords: Equity premium, idiosyncratic risk, higher-order risk aversion.

JEL classification: D01, D50, D53

In an economy with aggregate risk, the equity premium is the difference between the expected return of a dollar invested in an asset bearing the same risk as the whole economy and the risk-free interest rate. Equivalently, the equity premium measures how much more expensive

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a risk-less asset that pays the expected return of the economy is relative to the price of the risky return itself. This variable gained notoriety after Mehra and Prescott (1985) observed a significant difference between the empirical equity premium of the U.S. economy and its theoretical counterpart: the equity premium puzzle is the observation that standard macroeconomic models with homogeneous agents (both ex-ante and ex-post) fail to explain the equity premia typically observed in the data.¹

Shortly after, Mankiw (1986) presented a setting where ex-post heterogeneity affects the equity premium predicted by homogeneous agent models. Mankiw first observed that, in the presence of uninsurable idiosyncratic risk, how Mehra and Prescott (1985) modeled the equity premium is a misspecification, except under the assumption that all the agents in the economy have quadratic preferences.² Weil (1992) extended this insight by showing that higher-order derivatives of the agents' Bernoulli utility function matter for the determination of the effects of homoskedastic idiosyncratic risk on asset prices and premia. Later, Constantinides and Duffie (1996) again observed that failing to account for ex-post heterogeneity is akin to miss-estimating fundamentals such as the discount factor or the risk aversion coefficient. Indeed, they observed that the theoretical equity premium is higher in the case of CRRA preferences when there is counter-cyclically heteroskedastic idiosyncratic risk.³

Unfortunately, the magnitude and even the direction of the effects of ex-post heterogeneity on the equity premium depend on the details and assumptions of the model used to predict it theoretically. For the same class of preferences as Constantinides and Duffie (1996), Storesletten et al. (2007) showed that in an OLG economy, the effect of idiosyncratic shocks on the equity premium is significantly smaller than in the case considered by Constantinides and Duffie (1996). Moreover, Lettau (2002) argued that the economy's stochastic discount factor is independent of the volatility of idiosyncratic shocks when the agents have CRRA preferences and the ratio of the stochastic to the aggregate shocks is homoskedastic; and, through this mechanism, Krueger and Lustig (2010) showed that the result of Constantinides and Duffie (1996) does not hold when the agents in the economy have CRRA preferences and the distribution of idiosyncratic risk follows a particular form of pro-cyclical heteroskedasticity in a two-period economy. Under these assumptions, the equity premium is not affected by idiosyncratic risk.⁴

This paper aims to further our understanding of the effect of idiosyncratic risk on the equity

¹ See also Kocherlakota (1996) and Mehra (2003).

² Under these preferences, the agents don't demand savings for precautionary reasons, so the presence of idiosyncratic shocks does not affect the equilibrium prices of assets.

³ See also the empirical results in Cogley (2002).

⁴ See also Werning (2015) and Panageas et al. (2020).

premium. We consider different classes of preferences and different co-variations between the idiosyncratic shocks' variance and the economy's aggregate income. We offer a complete characterization of the effect for short-lived assets, such as those considered in Krueger and Lustig (2010), relying on the cross-moments of different utility function derivatives and the economy's aggregate income. A complete characterization is elusive for long-lived assets, such as those in Storesletten et al. (2007), but we present sufficient conditions for reversing the effect found by Constantinides and Duffie (1996).

We also study the effects of higher-order moments of the distribution of idiosyncratic risk. This exercise is motivated by the theoretical results of Martin (2013) and the empirical work of Guvenen et al. (2014) and Nakajima and Smirnyagin (2019), which highlight the importance of higher moments of the distribution of idiosyncratic shocks, in particular its negative skewness and high kurtosis, and of their cyclicalities.⁵

Our comparative statics results present a series of corrections to the theoretical equity premium using a parameterization of the moments of the distribution of idiosyncratic risk. Each correction relies on a Taylor expansion of the representative investor's marginal utility function, so a residual term is dismissed from the numerator and the denominator of the relative price of the risk-less to the risky asset. Our approach can be extended and applied in other contexts, but we recognize that no correction corresponds exactly to the equity premium except under extra assumptions. As a test of the robustness of our corrections, we compare the most primitive one to the exact premium in a simplified setting where the latter can be explicitly computed. The results suggest that the residual terms dismissed in our corrections are of second order.⁶

1 A TWO-PERIOD HOMOGENEOUS ECONOMY

Consider a society consisting of a unit mass of ex-ante identical individuals who live over two periods. In the present, each individual's wealth is the constant $\bar{w} > 0$. Their future wealth is the non-degenerate random variable W , whose support is a subset \mathcal{W} of \mathbb{R}_{++} .

The preferences of each individual over present consumption, c , and future risky consumption, C , are of the Selden type, namely represented by the function

$$v(c) + \beta v(u^{-1}(E[u(C)])), \quad (1)$$

where $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ and $\beta > 0$ capture the agent's preference for inter-temporal consumption

⁵ See also Kocherlakota (1997) and Barro (2006). Scott and Horvath (1980) and Kane (1982) study how the third and fourth moments of the distribution of risk affect investment decisions.

⁶ In an empirical exercise, Chabi-Yo and Loudis (2020) derives bounds for the market excess return by taking a Taylor expansion on the stochastic discount factors.

smoothing and her impatience, while $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is the agent's Bernoulli utility index, which models her attitude towards risk.⁷ Function v is $\mathbf{C}^1(\mathbb{R}_{++})$, strictly increasing and concave, whereas u is $\mathbf{C}^3(\mathbb{R}_{++})$, strictly increasing and strictly concave and has non-negative third derivative.

Only one asset can be traded: the asset that pays W in the second period. We interpret this asset as the “equity” of the economy.

1.1 Benchmark: only aggregate risk

In the absence of any other shocks, the present and future consumption of an individual in this economy are, respectively, $c = \bar{w} - q \cdot y$ and $C = W + W \cdot y$, where q denotes the price of the asset and y is the quantity of the asset demanded by the individual. The portfolio problem of each agent is, hence,

$$\max_y \left\{ v(\bar{w} - q \cdot y) + \beta v(u^{-1}(E[u(W + W \cdot y)])) \right\}.$$

Since all agents are identical, only a no-trade equilibrium is possible and

$$q = \frac{\beta v'(u^{-1}(E[u(W)]))}{v'(\bar{w})} \frac{E[u'(W) \cdot W]}{u'(u^{-1}(E[u(W)]))}. \quad (2)$$

If we define the function $m : \mathbb{R}_{++} \rightarrow \mathbb{R}$, as

$$m(w) = \frac{\beta v'(u^{-1}(E[u(W)]))}{v'(\bar{w})} \cdot \frac{E[u'(w)]}{u'(u^{-1}(E[u(W)]))},$$

then the economy's *stochastic discount factor* is the random variable $m(W)$, and we can re-write Eq. (2) as $q = E[m(W) \cdot W]$.

Using the same pricing kernel for other income flows, note that if the agents could also trade a risk-less asset with payoff $E(W)$, its price would equal

$$E[m(W) \cdot E(W)] = E[m(W)] \cdot E(W) = \frac{\beta v'(u^{-1}(E[u(W)]))}{v'(\bar{w})} \cdot \frac{E[u'(W)] \cdot E(W)}{u'(u^{-1}(E[u(W)]))}.$$

The *equity premium*, in the absence of any other risk, is

$$\bar{p} = \frac{E[m(W)] \cdot E(W)}{E[m(W) \cdot W]} - 1 = \frac{E[u'(W)] \cdot E(W)}{E[u'(W) \cdot W]} - 1 = -\frac{\text{Cov}[u'(W), W]}{E[u'(W) \cdot W]}, \quad (3)$$

where Cov is the *covariance operator*.⁸

⁷ See Selden (1978).

⁸ In the asset pricing literature, the risk-free rate r_f is defined by $1 + r_f = 1/E[m(W)]$, while the realized return of the market portfolio is the random variable $1 + R_m = W/q = W/E[m(W) \cdot W]$. The empirical equity premium equals

$$E(R_m - r_f) = \frac{E(W)}{E[m(W) \cdot W]} - \frac{1}{E[m(W)]} = \frac{1}{E[m(W)]} \cdot \left\{ \frac{E[m(W)] \cdot E(W)}{E[m(W) \cdot W]} - 1 \right\},$$

which implies that $E(R_m - r_f) = (1 + r_f)\bar{p}$. Our definition of the equity premium is, thus, the empirical equity premium discounted at the risk-free rate.

That the constant

$$\frac{\beta v'(u^{-1}(E[u(W)]))}{v'(\bar{w})} \cdot \frac{1}{u'(u^{-1}(E[u(W)]))}$$

cancels out in the computation of this relative price says that the equity premium depends on the agents' attitude towards risk but not on their impatience or attitude toward inter-temporal smoothing. While these two latter considerations affect the prices of the risky and the risk-less assets, their relative price depends *only* on the individual's attitude towards risk.⁹

1.2 Idiosyncratic risk

While all the agents in the economy are ex-ante identical, we want to consider the effects of ex-post heterogeneity. To model this, let random variable S , with $E(S | W) = 0$, be each agent's uninsurable, future idiosyncratic risk. When holding y units of the risky asset, an agent's future consumption is $C = W + S + W \cdot y$, and the equity premium is

$$p = \frac{E[m(C)] \cdot E(W)}{E[m(C) \cdot W]} - 1 = \frac{E[u'(C)] \cdot E(W)}{E[u'(C) \cdot W]} - 1. \quad (4)$$

Iterating expectations, this is

$$p = \frac{E\{E[u'(W + S) | W]\} \cdot E(W)}{E\{E[u'(W + S) | W] \cdot W\}} - 1. \quad (5)$$

Note from Eq. (5) that if the economy displays idiosyncratic risk, using Eq. (3) instead of Eq. (4) misspecifies the equity premium, as it amounts to assuming that

$$E[u'(W + S) | W] = u'(E(W + S | W)),$$

which in general requires that the Bernoulli function be quadratic.¹⁰

From now on, we assume that the individuals display strict prudence, namely that $u''' > 0$.

1.3 Idiosyncratic risk and the equity premium

Using the expansion

$$u'(w + s) \approx u'(w) + u''(w) \cdot s + \frac{1}{2} \cdot u'''(w) \cdot s^2, \quad (6)$$

we get that

$$E[u'(W + S) | W] \approx u'(W) + \frac{1}{2} \cdot u'''(W) \cdot \text{Var}(S | W), \quad (7)$$

⁹ The use of Selden preferences generalizes the more traditional Expected Utility approach: if one lets $v = u$, Eq. (1) becomes $u(c) + \beta E[u(C)]$. A similar argument to the previous observation yields that our results are valid in the case where the individual's preferences are represented by the function $v(c) + \beta E[u(C)]$.

¹⁰ This observation is Proposition 1 in Mankiw (1986) and part of the motivation for Weil (1992).

where Var is the variance operator. This observation allows us to propose a first correction to the equity premium: we approximate Eq. (5) by

$$\hat{p} = \frac{\mathbb{E} \left[u'(W) + \frac{1}{2} \cdot u'''(W) \cdot \text{Var}(S | W) \right] \cdot \mathbb{E}(W)}{\mathbb{E} \left\{ \left[u'(W) + \frac{1}{2} \cdot u'''(W) \cdot \text{Var}(S | W) \right] \cdot W \right\}} - 1, \quad (8)$$

and to perform comparative statics, we parameterize the conditional variance of the idiosyncratic shock by assuming that $\text{Var}(S | W) = \sigma^2 W^\eta$ almost surely, for constants $\sigma > 0$ and η .

The following result is the most basic one in the paper. Still, we argue it in detail, as the proofs of more involved results will resemble this argument.¹¹

THEOREM 1. *The equity premium \hat{p} ranges monotonically from \bar{p} , when $\sigma = 0$, to*

$$\lim_{\sigma \rightarrow \infty} \hat{p} = \frac{\mathbb{E} [u'''(W) \cdot W^\eta] \cdot \mathbb{E}(W)}{\mathbb{E} [u'''(W) \cdot W^{\eta+1}]} - 1.$$

Moreover, the following four statements are equivalent:

- (a) $\hat{p} \gtrless \bar{p}$;
- (b) $\frac{\partial \hat{p}}{\partial \sigma} \gtrless 0$;
- (c) $\frac{\mathbb{E} [u'''(W) \cdot W^\eta]}{\mathbb{E} [u'''(W) \cdot W^{\eta+1}]} \gtrless \frac{\mathbb{E} [u'(W)]}{\mathbb{E} [u'(W) \cdot W]}$; and
- (d) $\frac{\text{Cov}[u'''(W) \cdot W^\eta, W]}{\mathbb{E}[u'''(W) \cdot W^{\eta+1}]} \gtrless \frac{\text{Cov}[u'(W), W]}{\mathbb{E}[u'(W) \cdot W]}$.

Proof. Under the assumed functional form of $\text{Var}(S | W)$, Eq. (8) rewrites as

$$\hat{p} = \frac{\mathbb{E} [u'(W)] \cdot \mathbb{E}(W) + \frac{1}{2} \cdot \mathbb{E} [u'''(W) \cdot W^\eta] \cdot \mathbb{E}(W) \cdot \sigma^2}{\mathbb{E} [u'(W) \cdot W] + \frac{1}{2} \cdot \mathbb{E} [u'''(W) \cdot W^{\eta+1}] \cdot \sigma^2} - 1.$$

The two limits follow by direct computation.

The equivalence between (a) and (b) is straightforward. To see that (b) and (c) are equivalent, it simplifies our notation if we write

$$\hat{p} = \frac{N + \frac{1}{2} \cdot \mathbb{E} [u'''(W) \cdot W^\eta] \cdot \mathbb{E}(W) \cdot \sigma^2}{D + \frac{1}{2} \cdot \mathbb{E} [u'''(W) \cdot W^{\eta+1}] \cdot \sigma^2} - 1,$$

where $N = \mathbb{E} [u'(W)] \cdot \mathbb{E}(W)$ and $D = \mathbb{E} [u'(W) \cdot W]$ are, respectively, the numerator and the denominator in the definition of \bar{p} , as per Eq. (3). By direct computation, and since $\sigma > 0$, note that \hat{p} is increasing, constant or decreasing in σ depending on whether

$$D \cdot \mathbb{E}(W) \cdot \mathbb{E}[u'''(W) \cdot W^\eta] \gtrless N \cdot \mathbb{E}(W) \cdot \mathbb{E} [u'''(W) \cdot W^{\eta+1}].$$

¹¹ The theorem that follows continues to hold for any parameterization of the conditional variance of the form $\text{Var}(S | W = w) = \sigma^2 \cdot \eta(w)$, for any function $\eta : \mathcal{W} \rightarrow \mathbb{R}_{++}$. The subsequent results require the specific parameterization we are using.

By monotonicity and strict prudence, and since W takes only positive values, $D > 0$, $E(W) > 0$, and $E[u'''(W) \cdot W^{\eta+1}] > 0$. Thus, we can rewrite this expression as

$$\frac{E[u'''(W) \cdot W^\eta]}{E[u'''(W) \cdot W^{\eta+1}]} \geq \frac{N}{D},$$

namely statement (c).

Finally, to see that (c) and (d) are equivalent, note that

$$\begin{aligned} \frac{E[u'''(W) \cdot W^\eta]}{E[u'''(W) \cdot W^{\eta+1}]} &\geq \frac{E[u'(W)]}{E[u'(W) \cdot W]} \\ \Leftrightarrow \frac{E[u'''(W) \cdot W^\eta] \cdot E(W)}{E[u'''(W) \cdot W^{\eta+1}]} - 1 &\geq \frac{E[u'(W)] \cdot E(W)}{E[u'(W) \cdot W]} - 1 \\ \Leftrightarrow -\frac{\text{Cov}[u'''(W) \cdot W^\eta, W]}{E[u'''(W) \cdot W^{\eta+1}]} &\geq -\frac{\text{Cov}[u'(W), W]}{E[u'(W) \cdot W]} \end{aligned} \quad \square$$

Later on, it will be helpful to write condition (c) more concisely as

$$\frac{E[u'''(W) \cdot W^\eta] \cdot E(W)}{E[u'''(W) \cdot W^{\eta+1}]} - 1 \geq \bar{p}.$$

Also, note that the denominators on both sides of the expression in condition (d) are positive, and risk aversion implies that the numerator on its left-hand side is negative, so the ratio on the left-hand side is negative. None of our assumptions so far pins down the sign of the numerator on the right-hand side, though.

2 TWO IMPORTANT EXAMPLES

We now consider two canonical classes of Bernoulli functions to obtain concrete results.

2.1 CARA Preferences

To study the class of functions that display constant absolute risk aversion, in this section, we assume that the Bernoulli function is exponential, namely that $u(w) = -e^{-\alpha w}$ for some constant $\alpha > 0$.

THEOREM 2. *Suppose that u displays CARA. Whether the equity premium is larger, equal, or smaller in the presence of idiosyncratic risk depends on whether this risk is counter-, a-, or pro-cyclical. That is,*

$$\hat{p} \geq \bar{p} \Leftrightarrow \eta \leq 0.$$

Proof. We know from Theorem 1, by direct computation, that under this functional form,

$$\hat{p} \geq \bar{p} \Leftrightarrow \frac{E(\alpha^3 e^{-\alpha W} \cdot W^\eta)}{E(\alpha^3 e^{-\alpha W} \cdot W^{\eta+1})} \geq \frac{E(\alpha e^{-\alpha W})}{E(\alpha e^{-\alpha W} \cdot W)}.$$

Whether \hat{p} is larger, equal, or smaller than \bar{p} depends thus on the sign of

$$E(e^{-\alpha W} \cdot W^\eta) \cdot E(e^{-\alpha W} \cdot W) - E(e^{-\alpha W}) \cdot E(e^{-\alpha W} \cdot W^{\eta+1}).$$

If we let V be an (ancillary) random variable distributed identically to W and independent from it, we can rewrite the latter expression as $E[e^{-\alpha(W+V)} \cdot W^\eta \cdot (V - W)]$, which is proportional by a factor of $1/2 \Pr(V \neq W) > 0$, to

$$E[e^{-\alpha(W+V)} \cdot W^\eta \cdot (V - W) \mid V > W] + E[e^{-\alpha(W+V)} \cdot W^\eta \cdot (V - W) \mid V < W].$$

This expression is equivalent to

$$E[e^{-\alpha(W+V)} \cdot W^\eta \cdot (V - W) \mid V > W] + E[e^{-\alpha(V+W)} \cdot V^\eta \cdot (W - V) \mid W < V],$$

which, by direct computation, is

$$E[e^{-\alpha(W+V)} \cdot (W^\eta - V^\eta) \cdot (V - W) \mid V > W].$$

This number is positive, null, or negative, depending on whether η is negative, null, or positive: the terms $e^{-\alpha(W+V)}$ and $V - W$ are positive, given the condition of the expectation; the integrand, thus, has the same sign as $W^\eta - V^\eta$ when $V > W$. \square

Note that if $\eta = 0$, the argument is pretty simple:

$$\hat{p} = \frac{E(\alpha^3 e^{-\alpha W})}{E(\alpha^3 e^{-\alpha W} \cdot W)} = \frac{E(\alpha e^{-\alpha W})}{E(\alpha e^{-\alpha W} \cdot W)} = \bar{p}.$$

2.2 CRRA preferences

We now focus on Bernoulli functions with constant relative risk aversion. The property that this class gives us is that the first derivative of the Bernoulli function is homogeneous so that we can write $u'(w) = u'(1) \cdot w^{-\rho}$ for some constant $\rho > 0$.

THEOREM 3. *Suppose that u displays CRRA. Whether the equity premium is larger, the same, or smaller in the presence of idiosyncratic risk depends on whether η is smaller, equal, or larger than 2. That is,*

$$\hat{p} \begin{matrix} \geq \\ \leq \end{matrix} \bar{p} \Leftrightarrow \eta \begin{matrix} \leq \\ \geq \end{matrix} 2.$$

Proof. Substituting the functional form of the conditional variance of S , we get, again by Theorem 1, that

$$\hat{p} \begin{matrix} \geq \\ \leq \end{matrix} \bar{p} \Leftrightarrow \frac{E[\rho(1 + \rho) \cdot W^{\eta-\rho-2}]}{E[\rho(1 + \rho) \cdot W^{\eta-\rho-1}]} \begin{matrix} \geq \\ \leq \end{matrix} \frac{E(W^{-\rho})}{E(W^{-\rho+1})}.$$

We thus need to show that $\eta < 2$ is necessary and sufficient for

$$E(W^{\eta-\rho-2}) \cdot E(W^{-\rho+1}) > E(W^{-\rho}) \cdot E(W^{\eta-\rho-1}).$$

Let us define the random variable V as in the proof of Theorem 2. By direct computation, we need to argue that

$$\mathbb{E} [W^{-\rho} \cdot V^{-\rho+1} \cdot (W^{\eta-2} - V^{\eta-2})] > 0.$$

Using the same technique as in the proof of Theorem 2, the left-hand side of this expression is directly proportional to

$$\mathbb{E} [W^{-\rho} \cdot V^{-\rho} \cdot (V^{\eta-2} - W^{\eta-2}) \cdot (W - V) \mid V > W].$$

This expression is positive if, and only if, $\eta < 2$. □

As before, note that if $\text{Var}(S \mid W) = \sigma^2 W^2$, with $u'''(w) = \rho(\rho + 1)u(1) \cdot w^{-(\rho+2)}$, we get

$$\hat{p} = \frac{\mathbb{E} \left\{ \left[1 + \rho(\rho + 1) \frac{\sigma^2}{2} \right] \cdot W^{-\rho} \right\} \cdot \mathbb{E}(W)}{\mathbb{E} \left\{ \left[1 + \rho(\rho + 1) \frac{\sigma^2}{2} \right] \cdot W^{-\rho+1} \right\}} - 1 = \bar{p}.$$

2.3 Risk aversion and the cyclicalities of the volatility of idiosyncratic shocks

A comparison of the previous two theorems suggests a connection between the behavior of the risk aversion coefficients, the behavior of the conditional variance of the idiosyncratic shocks, and the latter's effect on the equity premium.

As is well known, the absolute risk aversion coefficient approximates the willingness to pay to insure against *additive* shocks of variance 2, and Theorem 2 states that when such willingness to pay is constant, the equity premium increases, remains, or decreases, depending on whether the volatility of the *absolute* idiosyncratic shock S is counter-cyclical, acyclical, or pro-cyclical.

The relative risk aversion coefficient, on the other hand, approximates an agent's willingness to pay to insure against *multiplicative* shocks of variance 2. In the class of preferences of Theorem 3, this coefficient is constant, and we can rewrite the conclusion of that theorem as saying that the presence of idiosyncratic risk increases, decreases, or leaves the risk premium unchanged depending on whether the volatility of the *relative* idiosyncratic shock S/W is counter-cyclical, acyclical, or pro-cyclical.

To reiterate, in particular:

COROLLARY 1. *Suppose that the absolute idiosyncratic shock is homoskedastic so that $\text{Var}(S \mid W) = \sigma^2 > 0$ almost surely. Then:*

- (a) *if the Bernoulli function exhibits CARA, \hat{p} does not depend on σ ; while*
- (b) *if it displays CRRA, \hat{p} is increasing in σ .*

If, alternatively, the relative idiosyncratic shock is homoskedastic and $\text{Var}(S/w \mid W) = \sigma^2 > 0$ almost surely, then:

(c) if the Bernoulli function displays CARA, \hat{p} is decreasing in σ ; while

(d) if it is of CRRA, \hat{p} does not depend on σ .

2.4 Accuracy of the approximations

The correction we propose to the equity premium relies on Taylor approximations to the agents' marginal utility function. Equations (6) and (7) are equivalent to

$$u'(w + s) = u'(w) + u''(w) \cdot s + \frac{1}{2} \cdot u'''(w) \cdot s^2 + O(s^3),$$

and

$$\mathbb{E}[u'(W + S) \mid W] = u'(W) + \frac{1}{2} \cdot u'''(W) \cdot \text{Var}(S \mid W) + o(\text{Var}(S \mid W)),$$

so in defining \hat{p} we dismiss $o(\text{Var}(S \mid W))$ terms in both its numerator and its denominator. To assess the quantitative implication of this dismissal, we consider an example where we can compute the equity premium p exactly, and compare it with \hat{p} .

Suppose the aggregate income W takes two values, w_h and w_ℓ , with equal probability. In addition, suppose the idiosyncratic shock S follows the uniform distribution over the interval $[-\delta_i, \delta_i]$, conditional on aggregate state $i = h, \ell$. Under these assumptions,

$$\mathbb{E}[u'(W + S) \mid W = w_i] = \int_{-\delta_i}^{\delta_i} \frac{u'(W + s)}{2\delta_i} ds = \frac{u(w_i + \delta_i) - u(w_i - \delta_i)}{2\delta_i}. \quad (9)$$

Tables 1 and 2 report the values of \bar{p} , p , and \hat{p} , namely the equity premium when ignoring idiosyncratic risk, the actual premium, and the correction of the former by the variance of the idiosyncratic component. In those tables, $w_h = 10$ and $w_\ell = 8$, while the other parameters are allowed to vary.

To measure idiosyncratic volatility, we calibrate σ so that the ratio

$$\frac{\min\{W + S\}}{\min\{W\}}$$

takes the values 95%, 90%, 75%, and 50%. We consider both classes of canonical Bernoulli functions and different values of their respective risk aversion coefficients. Importantly, we consider three values of η for each class that make the absolute or relative shock's volatility counter-cyclical, a-cyclical, or pro-cyclical.

Table 1 shows the results for CARA Bernoulli functions. In these computations, the differences between \hat{p} and p are always small, which brings confidence about the conclusions we derive

from our theorems. In line with Theorem 2, when the variance of S is acyclical, so $\eta = 0$, $\hat{p} = p = \bar{p}$. When such variance has cyclical variation, the qualitative results of the theorem also hold, and, importantly, the table gives us that

$$\hat{p} \geq p \geq \bar{p} \Leftrightarrow \eta \leq 0.$$

When approximating the economy's stochastic discount factor as we have, our measures of the equity premium need not be equal to the actual premium of the economy, and we only consider them to be corrections to the premium computed when idiosyncratic risk is ignored. Still, the direction of our corrections agrees with the actual effect of the risk, and \hat{p} is a much better estimate of p than \bar{p} .

For the case of CRRA preferences, we report the results in Table 2. When $\eta = 2$, the relative shock S/W is homoskedastic, and the table confirms the result of Theorem 3: $\hat{p} = p = \bar{p}$. When $\eta = 1$ or $\eta = 3$, the variance of S/W is, respectively, counter- or pro-cyclical. In these cases, the result also conforms with the insight of the theorem and, moreover,

$$p \geq \hat{p} \geq \bar{p} \Leftrightarrow \eta \leq 2.$$

Once again, the direction of our corrections conforms with the actual effect, and the difference between \hat{p} and p is significantly smaller than between \bar{p} and p .

3 SKEWNESS

The equity premium defined in Eq. (8) is a correction to Eq. (3) that takes into account the effects of the variance of idiosyncratic risk *only*. Guvenen et al. (2014), however, noted the significance of the latter risk's negative skewness. This section proposes a further correction to the premium that considers this moment. We determine the effect of skewness on the premium, re-calculate the effect of the variance, and calculate how the skewness changes the magnitude of the latter effect.

Our previous analysis relied on the second-order expansion (6). From now on, we refer to the premium resulting from this expansion, namely Eq. (8), as \hat{p}_2 . The third-order expansion

$$u'(w + s) \approx u'(w) + u''(w) \cdot s + \frac{1}{2} \cdot u'''(w) \cdot s^2 + \frac{1}{6} \cdot u^{[4]}(w) \cdot s^3$$

yields

$$\mathbb{E}[u'(W + S) | W] \approx u'(W) + \frac{1}{2} \cdot u'''(W) \cdot \text{Var}(S | W) + \frac{1}{6} \cdot u^{[4]}(W) \cdot \mathbb{E}(S^3 | W).$$

3.1 Acyclical skewness

Assume first that the skewness coefficient of the distribution of idiosyncratic risk is the constant

$$\gamma = \text{Skew}(S | W) = \frac{\mathbb{E}(S^3 | W)}{\text{Var}(S | W)^{3/2}} \leq 0, \quad (10)$$

and maintain the assumption that $\text{Var}(S | W) = \sigma^2 W^\eta$. In this setting, we define the premium

$$\hat{p}_3 = \frac{\mathbb{E} \left[u'(W) + \frac{\sigma^2}{2} \cdot u'''(W) \cdot W^\eta + \frac{\gamma \sigma^3}{6} \cdot u^{[4]}(W) \cdot W^{3\eta/2} \right] \cdot \mathbb{E}(W)}{\mathbb{E} \left\{ \left[u'(W) + \frac{\sigma^2}{2} \cdot u'''(W) \cdot W^\eta + \frac{\gamma \sigma^3}{6} \cdot u^{[4]}(W) \cdot W^{3\eta/2} \right] \cdot W \right\}} - 1,$$

which explicitly corrects for the effect of skewness and redefines the effect of the variance on the equity premium.

To ensure positive asset prices, we further assume that the Bernoulli function is *temperate*, or risk-averse of order 4, namely that $u^{[4]} \leq 0$.¹² Our first result is on the direct effect of the skewness on the premium:

THEOREM 4. *The equity premium \hat{p}_3 ranges monotonically between \hat{p}_2 , when $\gamma = 0$, and*

$$\lim_{\gamma \rightarrow -\infty} \hat{p}_3 = \frac{\mathbb{E} [u^{[4]}(W) \cdot W^{3\eta/2}] \cdot \mathbb{E}(W)}{\mathbb{E} [u^{[4]}(W) \cdot W^{1+3\eta/2}]} - 1.$$

Moreover, the following three statements are equivalent:

- (a) $\hat{p}_3 \geq \hat{p}_2$;
- (b) $\frac{\partial \hat{p}_3}{\partial \gamma} \geq 0$; and
- (c) $\frac{\mathbb{E} [u^{[4]}(W) \cdot W^{3\eta/2}] \cdot \mathbb{E}(W)}{\mathbb{E} [u^{[4]}(W) \cdot W^{1+3\eta/2}]} - 1 \geq \hat{p}_2$.

The proof of this result is very similar to the argument for Theorem 1, so we defer it, along with all the other proofs in the paper, to an appendix.¹³

Our next result re-calculates the effect of the variance, given the skewness:

THEOREM 5. *The equity premium \hat{p}_3 ranges between \bar{p} , when $\sigma = 0$, and*

$$\lim_{\sigma \rightarrow \infty} \hat{p}_3 = \frac{\mathbb{E} [u^{[4]}(W) \cdot W^{3\eta/2}] \cdot \mathbb{E}(W)}{\mathbb{E} [u^{[4]}(W) \cdot W^{1+3\eta/2}]} - 1.$$

Moreover, $\hat{p}_3 \geq \bar{p}$ and $\partial \hat{p}_3 / \partial \sigma \geq 0$ if

$$\frac{\mathbb{E} [u^{[4]}(W) \cdot W^{3\eta/2}]}{\mathbb{E} [u^{[4]}(W) \cdot W^{1+3\eta/2}]} \geq \frac{\mathbb{E} [u'''(W) \cdot W^\eta]}{\mathbb{E} [u'''(W) \cdot W^{1+\eta}]} \geq \frac{\mathbb{E} [u'(W)]}{\mathbb{E} [u'(W) \cdot W]}. \quad (11)$$

¹² See Eeckhoudt and Schlesinger (2006).

¹³ A condition analogous to (d) in Theorem 1 can be derived, but it is rather cumbersome and uninformative.

If both of these inequalities fail, then $\hat{p}_3 \leq \bar{p}$ and $\partial \hat{p}_3 / \partial \sigma \leq 0$.

Finally, we compute how the idiosyncratic shock's skewness affects the way its variance impacts the premium.

THEOREM 6. *Suppose first that Eq. (11) holds. Then,*

(a) $\frac{\partial^2 \hat{p}_3}{\partial \gamma \partial \sigma^2} \geq 0$ if

$$\gamma \leq \min \left\{ \frac{\mathbb{E}[u'(W) \cdot W] + \frac{\sigma^2}{2} \cdot \mathbb{E}[u'''(W) \cdot W^{\eta+1}]}{\frac{\sigma^3}{6} \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2+1}]}, \frac{3\mathbb{E}[u'''(W) \cdot W^{\eta+1}]}{\sigma^4 \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2+1}]} \right\}; \quad (12)$$

(b) $\frac{\partial^2 \hat{p}_3}{\partial \gamma \partial \sigma^2} \leq 0$ if

$$\gamma \geq \max \left\{ \frac{\mathbb{E}[u'(W) \cdot W] + \frac{\sigma^2}{2} \cdot \mathbb{E}[u'''(W) \cdot W^{1+\eta}]}{\frac{\sigma^3}{6} \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{1+3\eta/2}]}, \frac{3\mathbb{E}[u'''(W) \cdot W^{1+\eta}]}{\sigma^4 \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{1+3\eta/2}]} \right\}. \quad (13)$$

On the other hand, if both inequalities in Eq. (11) fail, then the condition in Eq. (12) suffices for $\partial^2 \hat{p}_3 / \partial \gamma \partial \sigma^2 \leq 0$, while the condition in Eq. (13) implies that $\partial^2 \hat{p}_3 / \partial \gamma \partial \sigma^2 \geq 0$.

3.2 The two examples

These theorems allow us to determine the effects unambiguously for the two canonical families of Bernoulli functions studied in Section 2. For concreteness and to maintain simpler expressions, we consider only the cases when the absolute and relative idiosyncratic shock is homoskedastic.

Since the exponential Bernoulli function of Subsection 2.1 is temperate, we can apply the previous theorems:

THEOREM 7. *Suppose that u displays CARA.*

(a) *If the absolute idiosyncratic shock is homoskedastic, namely if $\eta = 0$, then \hat{p}_3 depends neither on the variance σ nor on the skewness γ . In fact, $\hat{p}_3 = \hat{p}_2 = \bar{p}$.*

(b) *If, on the other hand, the relative idiosyncratic shock is homoskedastic, so $\eta = 2$, then \hat{p}_3 is decreasing in σ and increasing in γ , and $\hat{p}_3 \leq \hat{p}_2 < \bar{p}$. Also,*

$$\frac{\partial^2 \hat{p}_3}{\partial \gamma \partial \sigma^2} \geq (\leq) 0$$

for

$$\gamma \geq \max (\leq \min) \left\{ -\frac{\mathbb{E}[e^{-\alpha W} \cdot W \cdot (1 + \alpha^2 \sigma^2 / 2 W^2)]}{\alpha^3 \sigma^3 \cdot \mathbb{E}(e^{-\alpha W} \cdot W^4)}, -\frac{3 \cdot \mathbb{E}(e^{-\alpha W} \cdot W^3)}{\alpha \sigma^4 \cdot \mathbb{E}(e^{-\alpha W} \cdot W^4)} \right\}.$$

As for the CRRA preferences of Subsection 2.2, since they too are temperate:

THEOREM 8. *Suppose that u displays CRRA.*

- (a) *If the absolute idiosyncratic shock is homoskedastic, so $\eta = 0$, then \hat{p}_3 is increasing in σ and decreasing in γ , and $\hat{p}_3 \geq \hat{p}_2 > \bar{p}$. Moreover,*

$$\frac{\partial^2 \hat{p}_3}{\partial \gamma \partial \sigma^2} \geq (\leq) 0$$

whenever

$$\gamma \leq \min (\geq \max) \left\{ -\frac{6 \cdot [\mathbb{E}(W^{-\rho+1}) + \sigma^2/2 \cdot (\rho+1) \cdot \mathbb{E}(W^{-\rho-1})]}{\sigma^3 \cdot (\rho+1) \cdot (\rho+2) \cdot \mathbb{E}(W^{-\rho-2})}, -\frac{3 \cdot \mathbb{E}(W^{-\rho-1})}{\sigma^4 \cdot (\rho+2) \mathbb{E}(W^{-\rho-2})} \right\}.$$

- (b) *If, on the other hand, the relative idiosyncratic shock is homoskedastic, namely if $\eta = 2$, then \hat{p}_3 depends neither on the variance σ nor on the skewness γ and $\hat{p}_3 = \hat{p}_2 = \bar{p}$.*

3.3 Cyclicalities of the skewness

In the previous two subsections, we maintained that the skewness of the idiosyncratic shock, conditional on the aggregate income, is negative and constant as per Eq. (10). Under our parameterization for the conditional variance, this means that the third moment $\mathbb{E}(S^3 | W) = \gamma \sigma^3 W^{3\eta/2}$ is pro-cyclical when the variance is counter-cyclical, namely when $\eta > 0$. While this feature is consistent with the findings of Nakajima and Smirnyagin (2019),¹⁴ we now complement the analysis by allowing for an extra parameter that disentangles the dependence on the two moments on the realization of aggregate income.

Maintaining, hence, the assumption that $\text{Var}(S | W) = \sigma^2 W^\eta$, we further assume that $\text{Skew}(S | W) = \gamma W^\zeta$ almost surely. This amounts to the assumption that $\mathbb{E}(S^3 | W) = \gamma \sigma^3 W^{\zeta+3\eta/2}$, which is counter-cyclical if, and only if, $\zeta + 3\eta/2 < 0$.¹⁵

Under this assumption, Theorem 4 continues to be valid so long as we rewrite condition (c) as

$$\frac{\mathbb{E}[u^{[4]}(W) \cdot W^{\zeta+3\eta/2}] \cdot \mathbb{E}(W)}{\mathbb{E}[u^{[4]}(W) \cdot W^{1+\zeta+3\eta/2}]} - 1 \stackrel{\leq}{\geq} \hat{p}_2,$$

and Theorem 5 simply requires that Eq. (11) be adapted as

$$\frac{\mathbb{E}[u^{[4]}(W) \cdot W^{\zeta+3\eta/2}]}{\mathbb{E}[u^{[4]}(W) \cdot W^{1+\zeta+3\eta/2}]} \geq \frac{\mathbb{E}[u'''(W) \cdot W^\eta]}{\mathbb{E}[u'''(W) \cdot W^{1+\eta}]} \geq \frac{\mathbb{E}[u'(W)]}{\mathbb{E}[u'(W) \cdot W]}.$$

Regarding the effects of the two coefficients σ and γ , the presence of the two parameters η and ζ requires that a joint condition be satisfied and leaves room for some ambiguity. When the

¹⁴ See also Catherine (2022).

¹⁵ We maintain the assumption that $\gamma \leq 0$.

Bernoulli function exhibits CARA,

$$\hat{p}_3 \geq \hat{p}_2 \geq \bar{p} \Leftarrow \zeta \leq \min \left\{ -\frac{3\eta}{2}, -\frac{\eta}{2} \right\},$$

and

$$\hat{p}_3 \leq \hat{p}_2 \leq \bar{p} \Leftarrow \zeta \geq \max \left\{ -\frac{3\eta}{2}, -\frac{\eta}{2} \right\}.$$

For CRRA preferences, on the other hand,

$$\hat{p}_3 \geq \hat{p}_2 \geq \bar{p} \Leftarrow \zeta \leq \min \left\{ 3 - \frac{3\eta}{2}, 1 - \frac{\eta}{2} \right\},$$

and

$$\hat{p}_3 \leq \hat{p}_2 \leq \bar{p} \Leftarrow \zeta \geq \max \left\{ 3 - \frac{3\eta}{2}, 1 - \frac{\eta}{2} \right\}.$$

These last observations generalize the corresponding parts of Theorems 7 and 8. Statement (a) in Theorem 7 covers the case when the absolute idiosyncratic shock S has acyclical conditional second and third moments, which corresponds to $\zeta = 3\eta/2 = \eta/2$. Interestingly, statement (b) in Theorem 8 imposes the same acyclicity on the conditional moments of the relative shock S/W : with $\text{Var}(S/W \mid W) = \sigma^2 W^{\eta-2}$ and $E[(S/W)^3 \mid W] = \gamma \sigma^3 W^{\zeta+3\eta/2-3}$, acyclicity of these two moments requires $\eta = 2$ and $\zeta = 0$, which is precisely where $\zeta = 3 - 3\eta/2 = 1 - \eta/2$.

4 HIGHER-ORDER MOMENTS

The effect of higher-order moments of the distribution of idiosyncratic shocks on the equity premium critically depends on the agents' higher-order risk aversion.¹⁶ In this section, we provide a full characterization using the n^{th} -order expansion on the distribution of the marginal utility.

Suppose that the i^{th} standardized moment of the conditional distribution of S is the constant $\mu_i < \infty$ almost surely.¹⁷ The n^{th} -order approximation to the marginal utility is, analogously to Eq. (6),

$$u'(w + s) \approx \sum_{i=1}^{n+1} \frac{1}{(i-1)!} \cdot u^{[i]}(w) \cdot s^{i-1}.$$

This expression yields a further correction to the equity premium,

$$\hat{p}_n = \frac{E \left[\sum_{i=1}^{n+1} \frac{1}{(i-1)!} \cdot u^{[i]}(W) \cdot \text{Var}(S \mid W)^{\frac{i-1}{2}} \cdot \mu_{i-1} \right] \cdot E(W)}{E \left[\sum_{i=1}^{n+1} \frac{1}{(i-1)!} \cdot u^{[i]}(W) \cdot \text{Var}(S \mid W)^{\frac{i-1}{2}} \cdot \mu_{i-1} \cdot W \right]} - 1, \quad (14)$$

¹⁶ Eeckhoudt and Schlesinger (2006) show that the sign of the n^{th} -order derivative of the utility function characterizes agent's n^{th} order risk attitude. In addition, Loubergé et al. (2020) extends the result to multiplicative risks.

¹⁷ That is, that $E(S^i \mid W) = \text{Var}(S \mid W)^{i/2} \cdot \mu_i$, W -a.s., for all $i \geq 2$. We maintain the assumption that $\mu_1 = 0$, and adopt the convention that $\mu_0 = 1$.

which we can use to study the effects of the higher-order moments in the same way we used Eq. (8) to study the effect of its variance.

Following Eeckhoudt et al. (1995), we will say that the Bernoulli function u is *risk-averse of order n* if $(-1)^i \cdot u^{[i]} < 0$ for all $i = 1, \dots, n$.¹⁸ This property strengthens the usual hypothesis that u was strictly non-satiated, strictly risk-averse, and strictly prudent.

Our following result characterizes the effect of each standardized moment μ_n on \hat{p}_n and allows us to order the different measures of the premium. As before, we adopt the parameterization $V(S | W) = \sigma^2 W^\eta$, although this makes no significant difference in the following result.

THEOREM 9. *Let $n \geq 3$, and suppose $(-1)^i \cdot \mu_i \geq 0$ for all $i = 1, \dots, n$. If the Bernoulli function is risk-averse of order $n+1$, then the equity premium \hat{p}_n ranges monotonically between $\lim_{\mu_n \rightarrow 0} \hat{p}_n = \hat{p}_{n-1}$ and*

$$\lim_{\mu_n \rightarrow (-1)^{n+1} \infty} \hat{p}_n = \frac{\mathbb{E}[u^{[n+1]}(W) \cdot W^{n\eta/2}] \cdot \mathbb{E}(W)}{\mathbb{E}[u^{[n+1]}(W) \cdot W^{n\eta/2} \cdot W]} - 1.$$

Moreover, the following three statements are equivalent:

- (a) $\hat{p}_n \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \hat{p}_{n-1}$;
- (b) $\frac{\partial \hat{p}_n}{\partial \mu_n} \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0$; and
- (c) $(-1)^n \cdot \left\{ \frac{\mathbb{E}[u^{[n+1]}(W) \cdot W^{n\eta/2}] \cdot \mathbb{E}(W)}{\mathbb{E}[u^{[n+1]}(W) \cdot W^{n\eta/2} \cdot W]} - 1 \right\} \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} (-1)^n \cdot \hat{p}_{n-1}$.

This theorem allows us to determine the effects of higher-order moments. For example, since both families of preferences are risk-averse of order 5, or *edgy*:

1. In the case of a CARA Bernoulli function:

- (a) If the absolute idiosyncratic shock is homoskedastic, so if $\eta = 0$, then \hat{p}_3 does not depend on the kurtosis μ_4 . In fact, $\hat{p}_4 = \hat{p}_3 = \hat{p}_2 = \bar{p}$.
- (b) If, on the other hand, the relative idiosyncratic shock is homoskedastic, so $\eta = 2$, then \hat{p}_4 is decreasing in μ_4 and $\hat{p}_4 < \hat{p}_3 \leq \hat{p}_2 < \bar{p}$.

2. If, on the other hand, the function is of CRRA, then:

- (a) If the absolute idiosyncratic shock is homoskedastic, so $\eta = 0$, then \hat{p}_4 is increasing in μ_4 . Moreover, $\hat{p}_4 > \hat{p}_3 \geq \hat{p}_2 > \bar{p}$.

¹⁸ Risk aversion of order four is often referred to as *temperance*, while aversion of order five is called *edginess*.

- (b) If, alternatively, the relative idiosyncratic shock is homoskedastic, namely if $\eta = 2$, then \hat{p}_4 does not depend on the kurtosis μ_4 and $\hat{p}_4 = \hat{p}_3 = \hat{p}_2 = \bar{p}$.

5 LONG-LIVED ASSETS

Our analysis so far has assumed that the economy's equity only pays dividends. If, more realistically, capital is long-lived, we must adjust our analysis to consider the asset's resale value as part of its future return. For a specific application, consider the case of a stationary overlapping generations economy where individuals live for two periods and the only asset in the economy pays W , i.i.d., every period.

In every period, suppose that there is a unit mass of young individuals, each of whom has an endowment \bar{w} , and a unit mass of old individuals who each own a unit of an asset whose dividend is the random variable W we studied before. If the asset is long-lived, the old agents get to consume both the dividend W and the price q of the asset. The young generation, in turn, pays q per unit of the asset, anticipating a payoff of $W + q$ one period later.

For reasons that will be clear below, we need to consider a simpler class of preferences to study this problem. In what follows, we assume that an agent that consumes c when young and C when old has lifetime utility $c + E[u(C)]$, with a Bernoulli function u that displays temperance.

Our goal in this section is to further our understanding of the observation by Storesletten et al. (2007) that the positive effect of the variance of idiosyncratic risk on the equity premium found by Constantinides and Duffie (1996) largely disappears in an OLG economy. While we provide sufficient conditions for the premium to increase or decrease in that variance, comparing the magnitudes of the effects proved elusive in general. Our results still suggest the same insight as in Storesletten et al. (2007).

5.1 Benchmark: only aggregate risk

In the absence of any other risk, the problem of the young generation is

$$\max_y \{ \bar{w} - q \cdot y + E[u((W + q) \cdot y)] \},$$

where y represents, as before, the agent's demand for equity. The first-order condition of this problem is that

$$q = E[u'((W + q) \cdot y) \cdot (W + q)],$$

while market clearing requires that $y = 1$, so q is the solution to the equation

$$q = E[u'(W + q) \cdot (W + q)].$$

By the same arguments as before, a risk-less asset *with the same expected payoff* should be priced at

$$E[u'(W + q)] \cdot [E(W) + q],$$

so the relative price of the risk-less asset to the risky asset (minus 1) is again the *equity premium*:

$$\bar{p} = \frac{E[u'(W + q)] \cdot [E(W) + q]}{E[u'(W + q) \cdot (W + q)]} - 1. \quad (15)$$

5.2 Idiosyncratic risk

If the old generation faces idiosyncratic risk S , the premium is

$$p = \frac{E[u'(W + q + S)] \cdot [E(W) + q]}{E[u'(W + q + S) \cdot (W + q)]} - 1, \quad (16)$$

and using Eq. (15) amounts to assuming that

$$E[u'(W + q + S) \mid W] = u'(E[W + q + S \mid W]).$$

In the same vein as Eq. (6), the quadratic expansion

$$u'(w + q + s) \approx u'(w + q) + u''(w + q) \cdot s + \frac{1}{2} \cdot u'''(w + q) \cdot s^2$$

yields the approximation

$$\hat{p} = \frac{E \left[u'(W + q) + \frac{1}{2} \cdot u'''(W + q) \cdot \text{Var}(S \mid W) \right] \cdot [E(W) + q]}{E \left\{ \left[u'(W + q) + \frac{1}{2} \cdot u'''(W + q) \cdot \text{Var}(S \mid W) \right] \cdot (W + q) \right\}} - 1 \quad (17)$$

to the equity premium.

The problem would be a trivial extension of the previous results were it not for the dependence of q on the distribution of S via the equality

$$q = E[u'(W + q + S) \cdot (W + q)].$$

The purpose of this paper is not to develop the general comparative statics of this dependence but to determine how that dependence affects how the distribution of S impacts the equity premium.

5.3 Idiosyncratic risk and the equity premium

Maintain the parameterization $\text{Var}(S \mid W = w) = \sigma^2 w^\eta$ and, to establish unambiguous language, let us denote the right-hand side of Eq. (17) as the function

$$\Pi(q, \sigma) = \frac{E \left[u'(W + q) + \frac{\sigma^2}{2} \cdot u'''(W + q) \cdot W^\eta \right] \cdot [E(W) + q]}{E \left\{ \left[u'(W + q) + \frac{\sigma^2}{2} \cdot u'''(W + q) \cdot W^\eta \right] \cdot (W + q) \right\}} - 1.$$

In what follows, we will say that *the equity premium is decreasing in the price of the asset* if $\partial\Pi/\partial q < 0$, and that it is *partially decreasing in σ* if $\partial\Pi/\partial\sigma < 0$. When, on the other hand, we say that *the premium is decreasing in σ* , we will refer to the overall effect

$$\frac{d\hat{p}}{d\sigma} = \frac{\partial\Pi}{\partial q} \cdot q' + \frac{\partial\Pi}{\partial\sigma}, \quad (18)$$

where q' results from implicitly differentiating

$$q = E[u'(W + q + S) \cdot (W + q)]$$

with respect to σ .

THEOREM 10. *Suppose that the price is increasing in σ .*

1. *If the premium is decreasing in the price, then a necessary condition for the premium to be non-decreasing in σ is that*

$$\frac{\text{Cov}(u'(W + q), W)}{E(u'(W + q))} > \frac{\text{Cov}(u'''(W + q) \cdot W^\eta, W)}{E(u'''(W + q) \cdot W^\eta)}. \quad (19)$$

2. *If, on the other hand, the premium is non-decreasing in q , then Eq. (19) suffices for it to be increasing in σ .*

Since some of the expressions that follow are lengthy, we will sometimes write the random variable $u^{[n]}(W + q)$ simply as $U^{[n]}$. For example, Eq. (19) becomes

$$\frac{\text{Cov}(U', W)}{E(U')} > \frac{\text{Cov}(U''' \cdot W^\eta, W)}{E(U''' \cdot W^\eta)}.$$

The tension between the direct effect of q and the direct effect of σ arises when the former is negative. Instead of attempting a full characterization, we find sufficient conditions to resolve this tension:

THEOREM 11. *Suppose that the Bernoulli function is risk-averse of order four. The premium is decreasing in asset price q if*

$$\text{Cov}(U''' \cdot W^\eta, W) \leq 0 \quad (20)$$

and

$$\min \left\{ \frac{\text{Cov}(U', W)}{E(U')}, \frac{\text{Cov}(U''' \cdot W^\eta, W)}{E(U''' \cdot W^\eta)} \right\} \geq \max \left\{ \frac{\text{Cov}(U'', W)}{E(U'')}, \frac{\text{Cov}(U^{[4]} \cdot W^\eta, W)}{E(U^{[4]} \cdot W^\eta)} \right\}. \quad (21)$$

With these two previous results, we can determine how the fact that the equity is long-lived affects our results in Section 2. Ideally, we would like to compare the magnitude of $\partial\Pi/\partial\sigma$ when the asset is short-lived with the magnitude of $d\Pi/d\sigma$ when it is long-lived. This goal was impracticable for us, except when $\partial\Pi/\partial\sigma = 0$. In that case, we next prove that $d\Pi/d\sigma < 0$. As Storesletten et al. (2007), when the equity is long-lived, the response of the premium to changes in the variance is “more negative.”

5.4 Homoskedastic risk and CARA preferences

Consider first the case of homoskedastic idiosyncratic risk, with $\eta = 0$ and $\text{Var}(S \mid W) = \sigma^2$ almost surely on W , and suppose that the Bernoulli function is exponential.

We already observed that these preferences are temperate. We can also pin down the sign of the effect of σ on q , as follows:

LEMMA 1. *Suppose that u displays CARA. Then, the price of the asset is increasing in σ .*

Theorems 10 and 11 immediately imply the following two results for this case:

1. the premium is decreasing in asset price q if

$$\min \left\{ \frac{\text{Cov}(U', W)}{E(U')}, \frac{\text{Cov}(U''', W)}{E(U''')} \right\} \geq \max \left\{ \frac{\text{Cov}(U'', W)}{E(U'')}, \frac{\text{Cov}(U^{[4]}, W)}{E(U^{[4]})} \right\}.$$

2. if the premium is decreasing in asset price, then a necessary condition for it to be non-decreasing in σ is that

$$\frac{\text{Cov}(U', W)}{E(U')} > \frac{\text{Cov}(U''', W)}{E(U''')}.$$

(To be sure, note that the condition that $\text{Cov}[U''', W] \leq 0$, which specializes Eq. (20) to the case at hand, does not have to be assumed, as it is implied by the assumption that $u^{[4]} < 0$.)

These two insights imply the following result. The first of the two statements above holds under CARA preferences. As per the second statement, this implies a necessary condition known to be violated.

THEOREM 12. *Suppose the Bernoulli function displays CARA, and the absolute idiosyncratic shock is homoskedastic. Then, the equity premium \hat{p} decreases in σ .*

5.5 Homoskedastic relative risk and CRRA preferences

Consider now the case where $\text{Var}(S/w \mid W = w) = \sigma$, namely when $\eta = 2$, and suppose that the first derivative of the Bernoulli function is homogeneous.

As before, we already know that these preferences are temperate, but before we apply our general results to this case, we need to determine the effects of σ on the price of equity:

LEMMA 2. *Suppose that the Bernoulli function displays CRRA. Then, the price of the asset is increasing in σ .*

Unfortunately, the assumption that $\eta = 2$, which is necessary to make the relative shock S/W homoskedastic, introduces ambiguity in the sign of terms of the form

$$\text{Cov}[u^{[n]}(W + q) \cdot \text{Var}(S \mid W), W].$$

For $n = 3$, for example, the term $u^{[n]}(w + q)$ is decreasing in w , but $\text{Var}(S \mid W = w)$ is increasing; for $n = 4$, both terms are increasing but the first one is negative. With diffidence, we resolve these ambiguities by assuming that

$$q \leq \frac{\min\{\rho, 1\}}{2} \cdot \inf \mathcal{W}. \quad (22)$$

This assumption is imposed on an endogenous variable, but it seems unavoidable.¹⁹ Once the condition is imposed, the following is true:

LEMMA 3. *Suppose that the Bernoulli function displays CRRA and Eq. (22) holds. Then, the equity premium \hat{p} decreases in asset price q .*

These two insights imply the following result:

THEOREM 13. *Suppose that the Bernoulli function displays CRRA, the relative idiosyncratic shock is homoskedastic, and Eq. (22) holds. Then, the equity premium \hat{p} decreases in σ .*

The strategy for the proof is the same as in Theorem 12: under the assumptions of the theorem, Lemmas 2 and 3 give us that all the conditions that make Eq. (19) necessary for \hat{p} non-decreasing in σ are satisfied. We argue that, nonetheless, Eq. (19) itself fails.

6 CONCLUDING REMARKS

6.1 Our results in the context of existing literature

In this paper, we have tried to further our understanding of the effects of the presence of uninsurable idiosyncratic risk on the relative price of the equity of an economy. The motivation comes from the early work by Mankiw (1986) and Weil (1992) and the fact that various works have obtained different results because of different settings. Table 3 summarizes how the results in the literature relate to some of the results obtained in this paper.

Our first set of results focuses on the effects of the volatility of the idiosyncratic risk. After observing that the agents' prudence creates a mechanism for these effects, as Mankiw (1986) had pointed out, we emphasize that the cyclical nature of that volatility and the behavior of the agents' risk aversion make a difference. If the agents display constant absolute risk aversion and the absolute idiosyncratic shock is homoskedastic, the volatility of this shock does not affect

¹⁹ A simple example shows that the assumption is not overly restrictive. Consider an economy with two aggregate states, h and ℓ , with $w_h = 10$ and $w_\ell = 8$. The probability of state h is 0.6, and the idiosyncratic risk S follows the uniform distribution over $[-1, 1]$. The traders have CRRA preference with $\rho = 2$. With these parameters, the equilibrium price $q = 0.7205$, much less than the right-hand side of Eq. (22), which equals 4. With many other assumptions and parameters, the assumption still holds.

the equity premium; under these preferences, a counter-cyclical variance of the idiosyncratic shock is necessary and sufficient for the equity premium to be higher due to the presence of this shock. Suppose the ratio of the idiosyncratic and the aggregate shocks is homoskedastic. In that case, a similar conclusion applies, *mutatis mutandis*, when the preferences display constant relative risk aversion. The literature had observed this latter result in Weil (1992) and Krueger and Lustig (2010), so we see our results as complementary.

The second set of results relates to the effect of higher moments of the distribution of the idiosyncratic shocks. Again, how these moments affect the equity premium depends on the behavior of the agents' coefficients of risk aversion and the type of cyclical behavior followed by the volatility of the idiosyncratic shock. For example, consider this time the case of preferences displaying constant relative risk aversion. If the relative shock is homoskedastic, once again, the higher moments of the distribution of idiosyncratic risk are immaterial for the level of the equity premium, whereas if it is the absolute shock that is homoskedastic, a negatively skewed, leptokurtic distribution generates higher equity premium. These effects are driven by the fourth and fifth derivatives of the agent's utility function, which Weil (1992) had already noted. In that sense, our contribution is the identification of which features of the distribution of idiosyncratic shocks interact with which derivatives to impact the equity premium.

The previous results are obtained for a two-period economy in which, by design, the equity of the economy is short-lived and only pays dividends. To consider long-lived equity, where the future resale value of an asset naturally affects the present price, we adapt our analysis to a two-generation OLG economy, which allows us to compare our results with those in Storesletten et al. (2007). Here, the mathematics is more convoluted, and further assumptions are necessary. Both for CARA and CRRA preferences, we consider the case in which if the asset were short-lived, the variance of the idiosyncratic shock would not affect the equity premium. Our results are that in those same cases, the variance has a negative effect on the premium if the asset is long-lived. We conjecture that, in general, if the equity is long-lived, the derivative of the premium with respect to the variance is lower than when the asset is short-lived.

6.2 *Extension to an infinite-horizon economy*

In all these results, all the agents were ex-ante homogeneous, and we studied only the effect of ex-post heterogeneity.²⁰ Ex-ante homogeneity is useful because it simplifies the equilibrium trade in assets, hence their equilibrium price.²¹ Under specific assumptions, our results allow us to say something in the case of an infinite-horizon Markovian economy.

²⁰ In the OLG economy, all the *young* agents are homogeneous.

²¹ In a different paper, we progress on studying the same problem under ex-ante heterogeneity.

Let W and S represent the next period aggregate and idiosyncratic shocks, while \tilde{W} and \tilde{S} denote the present variables. Given $\tilde{W} = \tilde{w}$ and $\tilde{S} = \tilde{s}$, suppose that each the agent maximizes

$$v(\bar{w} - q \cdot y) + \beta v(u^{-1}(\mathbb{E}[u(W + S + W \cdot y) \mid \tilde{W} = \tilde{w}]))$$

over y , where $\bar{w} = \tilde{w}(1 + y_-) + \tilde{s}$ and y_- denotes the amount of asset the agent carried from the last period. Since this economy is recursive, suppose that the maximum of this problem defines $u(\bar{w})$, which is the *continuation value* of starting a period with wealth \bar{w} .

If we assume that function v is linear, we can maintain the feature that there is no asset trade at equilibrium, which means that the equity premium in the present is the random variable

$$\tilde{P} = \frac{\mathbb{E}[u'(W + S) \mid \tilde{W}] \cdot \mathbb{E}(W \mid \tilde{W})}{\mathbb{E}[u'(W + S) \cdot W \mid \tilde{W}]} - 1.$$

Our results generalize to almost sure effects on \tilde{P} in this setting. This observation is helpful if the assumption on function v is tenable, as it extends our analysis from Selden preferences to the framework of Epstein and Zin (1991).

APPENDIX: PROOFS

Proof of Theorem 4: The computation of the two limits and the equivalence of (a) and (b) are straightforward, so we only need to prove that (b) and (c) are equivalent.

As in the proof of Theorem 1, we can rewrite

$$\hat{p}_3 = \frac{N_2 + \frac{1}{6} \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2}] \cdot \mathbb{E}(W) \cdot \sigma^3 \gamma}{D_2 + \frac{1}{6} \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2} \cdot W] \cdot \sigma^3 \gamma} - 1,$$

where N_2 and D_2 are, respectively, the numerator and the denominator in the definition of \hat{p}_2 , as in Eq. (14). By direct computation, $\partial \hat{p}_3 / \partial \gamma \geq 0$ if, and only if,

$$\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2}] \cdot \mathbb{E}(W) \cdot D_2 \geq \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2+1}] \cdot N_2. \quad (23)$$

With $u^{[4]} < 0$, by assumption, and since $D_2 > 0$, it follows that

$$\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2} \cdot W] \cdot D_2 < 0.$$

Using this inequality, Eq. (23) is equivalent to

$$\frac{\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2}] \cdot \mathbb{E}(W)}{\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2} \cdot W]} \leq \frac{N_2}{D_2}. \quad \square$$

Proof of Theorem 5: Once again, the two limits are straightforward. For the second claim, by direct computation, $\partial \hat{p}_3 / \partial \sigma^2 \geq 0$ if, and only if, the sum of the following three terms is non-positive:

$$\begin{aligned} & \mathbb{E}[u'(W) \cdot W] \cdot \mathbb{E}[u'''(W) \cdot W^\eta] - \mathbb{E}[u'(W)] \cdot \mathbb{E}[u'''(W) \cdot W^{\eta+1}] \\ & \frac{\sigma \gamma}{2} \cdot \left\{ \mathbb{E}[u'(W) \cdot W] \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2}] - \mathbb{E}[u'(W)] \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2+1}] \right\} \\ & \frac{\sigma^{3/2} \gamma}{12} \cdot \left\{ \mathbb{E}[u'''(W) \cdot W^{\eta+1}] \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2}] - \mathbb{E}[u'''(W) \cdot W^\eta] \cdot \mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2+1}] \right\}. \end{aligned}$$

The first of these terms is non-positive if and only if,

$$\frac{\mathbb{E}[u'''(W) \cdot W^\eta]}{\mathbb{E}[u'''(W) \cdot W^{\eta+1}]} \geq \frac{\mathbb{E}[u'(W)]}{\mathbb{E}[u'(W) \cdot W]}.$$

Since $\sigma \gamma \leq 0$, the second term is non-positive if, and only if,

$$\frac{\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2}]}{\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2+1}]} \geq \frac{\mathbb{E}[u'(W)]}{\mathbb{E}[u'(W) \cdot W]}.$$

And since $\sigma^{3/2} \gamma \leq 0$, the third term is non-positive if, and only if,

$$\frac{\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2}]}{\mathbb{E}[u^{[4]}(W) \cdot W^{3\eta/2+1}]} \geq \frac{\mathbb{E}[u'''(W) \cdot W^\eta]}{\mathbb{E}[u'''(W) \cdot W^{\eta+1}]}$$

The two inequalities in Eq. (11) suffice for these last three conditions, and hence for the sum of the three terms to be non-positive. \square

Proof of Theorem 6: The argument is largely computational. Suppose one has the function

$$\phi(x, y) = \frac{a + bx + cx^{3/2}y}{d + ex + fx^{3/2}y},$$

with $a, b, d, e \geq 0$, $c, f \leq 0$, $x > 0$ and $y \leq 0$. By direct computation,

$$\frac{\partial^2 \phi}{\partial y \partial x}(x, y) \geq 0$$

if, and only if,

$$\frac{3}{2}(cd - af)(d + ex - fx^{3/2}y)x^{1/2} + \frac{1}{2}(ce - bf)(e - fx^2y)x^{5/2} \geq 0. \quad (24)$$

The two inequalities in Eq. (11) guarantee that $cd \leq af$ and $ce \leq bf$. To guarantee Eq. (24), it then suffices that $d + ex \leq fx^{3/2}y$ and $e \leq fx^2y$. Recalling that $f \leq 0$, Eq. (12) delivers these two inequalities.

When both inequalities in Eq. (11) hold, Eq. (24) fails if $d + ex \geq fx^{3/2}y$ and $e \geq fx^2y$, which amount to Eq. (13).

The rest of the argument is similar. □

Proof of Theorem 7: For this type of preferences,

$$\hat{p}_3 = \frac{\mathbb{E}[e^{-\alpha W} \cdot (1 + \frac{\sigma^2}{2}\alpha^2 W^\eta - \frac{\sigma^3}{6}\alpha^3\gamma W^{3\eta/2})] \cdot \mathbb{E}(W)}{\mathbb{E}\left[e^{-\alpha W} \cdot (1 + \frac{\sigma^2}{2}\alpha^2 W^\eta - \frac{\sigma^3}{6}\alpha^3\gamma W^{3\eta/2}) \cdot W\right]} - 1.$$

The first claim is straightforward: if $\eta = 0$, after canceling constants,

$$\hat{p}_3 = \frac{\mathbb{E}(e^{-\alpha W}) \cdot \mathbb{E}(W)}{\mathbb{E}(e^{-\alpha W} \cdot W)} - 1 = \hat{p}_2 = \bar{p}.$$

When $\eta = 2$, for the second claim, the argument resembles the proof of Theorem 2.

By Theorem 9, in order to show that \hat{p}_3 is increasing in γ , it suffices to argue that

$$\frac{\mathbb{E}[u^{[4]}(W) \cdot W^3] \cdot \mathbb{E}(W)}{\mathbb{E}[u^{[4]}(W) \cdot W^4]} - 1 < \hat{p}_2.$$

By direct computation, this is equivalent to

$$\mathbb{E}(e^{-\alpha W} \cdot W^3) \cdot \mathbb{E}\left[(1 + \frac{\sigma^2}{2}\alpha^2 W^2) \cdot e^{-\alpha W} \cdot W\right] - \mathbb{E}(e^{-\alpha W} \cdot W^4) \cdot \mathbb{E}\left[(1 + \frac{\sigma^2}{2}\alpha^2 W^2) \cdot e^{-\alpha W}\right] < 0.$$

Letting random variable V be i.i.d. with W , the left-hand side of this expression rewrites as

$$\mathbb{E}\left[e^{-\alpha(V+W)} \cdot (1 + \frac{\sigma^2}{2}\alpha^2 V) \cdot (V - W) \cdot W^3\right].$$

This number is proportional, by a factor of $1/2 \Pr(V \neq W) > 0$, to

$$\mathbb{E}\left\{e^{-\alpha(V+W)} \cdot \left[(W^3 - V^3) + \frac{\sigma^2}{2}\alpha^2 V^2 W^2 (W - V)\right] \cdot (V - W) \mid V > W\right\} < 0.$$

To re-determine the effect of σ , by Theorem 5, it suffices to show that

$$\frac{\mathbb{E}(e^{-\alpha W})}{\mathbb{E}(e^{-\alpha W} \cdot W)} \geq \frac{\mathbb{E}(e^{-\alpha W} \cdot W^2)}{\mathbb{E}(e^{-\alpha W} \cdot W^3)} \geq \frac{\mathbb{E}(e^{-\alpha W} \cdot W^3)}{\mathbb{E}(e^{-\alpha W} \cdot W^4)}.$$

For the first inequality, the same technique used in the previous theorems, with random variable V being i.i.d. with W , allows us to show that

$$\mathbb{E}(e^{-\alpha W} \cdot W^2) \cdot \mathbb{E}(e^{-\alpha W} \cdot W) - \mathbb{E}(e^{-\alpha W}) \cdot \mathbb{E}(e^{-\alpha W} \cdot W^3)$$

is proportional, by a positive factor, to

$$\mathbb{E}\left[e^{-\alpha(W+V)} \cdot (W^2 - V^2) \cdot (V - W) \mid W > V\right] \leq 0.$$

For the second inequality,

$$\mathbb{E}(e^{-\alpha W} \cdot W^3)^2 - \mathbb{E}(e^{-\alpha W} \cdot W^4) \cdot \mathbb{E}(e^{-\alpha W} \cdot W^2)$$

is directly proportional to

$$\mathbb{E} \left[e^{-\alpha(W+V)} \cdot W^2 \cdot V^2 \cdot (-W^2 + 2VW - V^2) \mid W > V \right] \leq 0.$$

This result on the cross derivative follows immediately from Theorem 6, given that both inequalities in Eq. (11) fail, as seen above. \square

Proof of Theorem 8: For the first claim, note that the proof of Theorem 3 implies that

$$\frac{\mathbb{E}(W^{-\rho+1})}{\mathbb{E}(W^{-\rho})} > \frac{\mathbb{E}(W^{-\rho-1})}{\mathbb{E}(W^{-\rho-2})}.$$

By the same argument,

$$\frac{\mathbb{E}(W^{-\rho-1})}{\mathbb{E}(W^{-\rho-2})} > \frac{\mathbb{E}(W^{-\rho-2})}{\mathbb{E}(W^{-\rho-3})}.$$

It follows that

$$\frac{\mathbb{E}[u'(W) \cdot W]}{\mathbb{E}[u'(W)]} > \frac{\mathbb{E}[u'''(W) \cdot W]}{\mathbb{E}[u'''(W)]} > \frac{\mathbb{E}[u^{[4]}(W) \cdot W]}{\mathbb{E}[u^{[4]}(W)]},$$

and hence that

$$\mathbb{E}[u'(W) \cdot W] \cdot \mathbb{E}[u^{[4]}(W)] > \mathbb{E}[u'(W)] \cdot \mathbb{E}[u^{[4]}(W) \cdot W]$$

and

$$\mathbb{E}[u'''(W) \cdot W] \cdot \mathbb{E}[u^{[4]}(W)] > \mathbb{E}[u'''(W)] \cdot \mathbb{E}[u^{[4]}(W) \cdot W].$$

Aggregating,

$$\left\{ \mathbb{E} \left[u'(W) + \frac{\sigma^2}{2} \cdot u'''(W) \right] W \right\} \cdot \mathbb{E} \left[u^{[4]}(W) \right] > \mathbb{E} \left[u'(W) + \frac{\sigma^2}{2} \cdot u'''(W) \right] \cdot \mathbb{E} \left[u^{[4]}(W) \cdot W \right],$$

which implies, by Theorem 4, that \hat{p}_3 is decreasing in γ .

To see that \hat{p}_3 is increasing in σ , we prove that condition (11) holds and invoke Theorem 5. To see that

$$\frac{\mathbb{E}[u'''(W) \cdot W^\eta]}{\mathbb{E}[u'''(W) \cdot W^{\eta+1}]} \geq \frac{\mathbb{E}[u'(W)]}{\mathbb{E}[u'(W) \cdot W]},$$

we need to argue that

$$\mathbb{E}(W^{\eta-\rho-2}) \cdot \mathbb{E}(W^{1-\rho}) - \mathbb{E}(W^{\eta-\rho-1}) \cdot \mathbb{E}(W^{-\rho}) \geq 0.$$

Using, as before, an ancillary random variable V that is i.i.d. with W , the latter expectation is directly proportional to

$$\mathbb{E} \left[W^{-\rho} \cdot V^{-\rho} \cdot (V - W) \cdot (W^{-1} - V - 1) \cdot (W^{-1} + V - 1) \mid W > V \right] \geq 0.$$

The sign of the cross derivative follows from Theorem (6), upon substitution, since we just proved that condition (11) holds in this case.

As for the second claim, by direct computation,

$$\hat{p}_3 = \frac{\mathbb{E}(W^{-\rho}) \cdot \mathbb{E}(W) \cdot [1 + \rho(\rho + 1) \frac{\sigma^2}{2} + \rho(\rho + 1)(\rho + 2) \frac{\sigma^{3/2}}{6} \gamma]}{\mathbb{E}(W^{-\rho+1}) \cdot [1 + \rho(\rho + 1) \frac{\sigma^2}{2} + \rho(\rho + 1)(\rho + 2) \frac{\sigma^{3/2}}{6} \gamma]} - 1 = \hat{p}_2. \quad \square$$

Proof of Theorem 9: This proof directly generalizes previous arguments, so we omit it. \square

Proof of Theorem 10: Recall Eq. (18). For the first result, note that the first summand on the right-hand side of the last expression is negative, so a necessary condition for the sum to be positive is that the second summand be positive. For the second result, under the assumptions, the first summand is non-negative, so the sum is positive if so is the second summand.

In both cases, all one needs is that $\partial\Pi/\partial\sigma > 0$. The proof that this inequality is equivalent to Eq. (19) resembles the argument for Theorem 1, so we omit it. \square

Proof of Theorem 11: We can write Eq. (17) as

$$\hat{p} = \frac{f(q) + \sigma^2 \cdot g(q)}{\varphi(q) + \sigma^2 \cdot \gamma(q)},$$

where

$$\begin{aligned} f(q) &= \mathbb{E}[u'(W + q)] \cdot [E(W) + q], \\ g(q) &= \frac{1}{2}\mathbb{E}[u'''(W + q) \cdot W^\eta] \cdot [E(W) + q], \\ \varphi(q) &= \mathbb{E}[u'(W + q) \cdot (W + q)] \end{aligned}$$

and

$$\gamma(q) = \frac{1}{2}\mathbb{E}[u'''(W + q) \cdot W^\eta \cdot (W + q)].$$

With this formulation, \hat{p} is decreasing in q if, and only if,

$$[f'(q) + \sigma^2 \cdot g'(q)] \cdot [\varphi(q) + \sigma^2 \cdot \gamma(q)] < [\varphi'(q) + \sigma^2 \cdot \gamma'(q)] \cdot [f(q) + \sigma^2 \cdot g(q)],$$

which holds if

$$f'(q) \cdot \varphi(q) < \varphi'(q) \cdot f(q) \tag{25}$$

$$f'(q) \cdot \gamma(q) \leq \varphi'(q) \cdot g(q) \tag{26}$$

$$g'(q) \cdot \varphi(q) \leq \gamma'(q) \cdot f(q) \tag{27}$$

$$g'(q) \cdot \gamma(q) \leq \gamma'(q) \cdot g(q). \tag{28}$$

Upon substitution, Eq. (25) is equivalent to

$$\{\mathbb{E}(U'')[\mathbb{E}(W) + q] + \mathbb{E}(U')\} \cdot \mathbb{E}[U' \cdot (W + q)] < \{\mathbb{E}[U'' \cdot (W + q)] + \mathbb{E}(U')\} \cdot \mathbb{E}(U') \cdot [\mathbb{E}(W) + q],$$

which is, by direct computation,

$$\{\mathbb{E}(U'') \cdot \mathbb{E}[U' \cdot (W + q)] - \mathbb{E}(U') \cdot \mathbb{E}[U'' \cdot (W + q)]\} \cdot [\mathbb{E}(W) + q] + \mathbb{E}(U') \cdot \text{Cov}(U', W) < 0. \quad (*)$$

Since $u' > 0$ and $u'' < 0$, we have that $\mathbb{E}(U') > 0$ and $\text{Cov}(U', W) < 0$, so it suffices that

$$\mathbb{E}(U'') \cdot \mathbb{E}[U' \cdot (W + q)] \leq \mathbb{E}(U') \cdot \mathbb{E}[U'' \cdot (W + q)],$$

for inequality (*) to hold, as $\mathbb{E}(W) + q > 0$. As in the proof of Theorem 1, this is equivalent to

$$\frac{\text{Cov}(U', W)}{\mathbb{E}(U')} \geq \frac{\text{Cov}(U'', W)}{\mathbb{E}(U'')},$$

which is one of the inequalities that are part of Eq. (21).

Similarly, Eq. (26) is equivalent to the requirement that the sum of

$$\{\mathbb{E}(U'') \cdot \mathbb{E}[U''' \cdot W^\eta \cdot (W + q)] - \mathbb{E}[U''' \cdot W^\eta] \cdot \mathbb{E}[U'' \cdot (W + q)]\} \cdot [\mathbb{E}(W) + q] \tag{**}$$

and

$$\mathbb{E}(U') \cdot \text{Cov}[U''' \cdot W^\eta, W] \tag{***}$$

be non-positive.

Since $u' > 0$ and $\text{Cov}[U''' \cdot W^\eta, W] \leq 0$, we have that the expression in (***) is non-positive. On the other hand, since $E(W) + q > 0$, for inequality (**) to hold it suffices that

$$E(U'') \cdot E[U''' \cdot W^\eta \cdot (W + q)] \leq E[U''' \cdot W^\eta] \cdot E[U'' \cdot (W + q)],$$

which is equivalent to

$$\frac{\text{Cov}[U''' \cdot W^\eta, W]}{E[U''' \cdot W^\eta]} \geq \frac{\text{Cov}(U'', W)}{E(U'')}.$$

For Eqs. (25) and (26) to hold true, it thus suffices that

$$\min \left\{ \frac{\text{Cov}(U', W)}{E(U')}, \frac{\text{Cov}[U''' \cdot W^\eta, W]}{E[U''' \cdot W^\eta]} \right\} \geq \frac{\text{Cov}(U'', W)}{E(U'')}.$$

By a virtually identical analysis, using that $u''' > 0$ and $u^{[4]} < 0$, one can prove that

$$\min \left\{ \frac{\text{Cov}(U', W)}{E(U')}, \frac{\text{Cov}[U''' \cdot W^\eta, W]}{E[U''' \cdot W^\eta]} \right\} \geq \frac{\text{Cov}[U^{[4]} \cdot W^\eta, W]}{E[U^{[4]} \cdot W^\eta]}$$

suffices for Eqs. (27) and (28) □

Proof of Lemma 1: Let $u(w) = -e^{-\alpha w}$, for some $\alpha > 0$. Then, $u^{[n]}(w) = (-\alpha)^n u(w)$ and

$$q = E[u'(W + q + S) \cdot (W + q)] = e^{-\alpha q} \cdot \{E[u'(W + S) \cdot W] + E[u'(W + S)] \cdot q\},$$

so

$$e^{\alpha q} = \frac{E[u'(W + S) \cdot W]}{q} + E[u'(W + S)].$$

This expression is transcendental, so we can *only* obtain q' by implicit differentiation:

$$\left\{ e^{\alpha q} + \frac{E[u'(W + S) \cdot W]}{q^2} \right\} \cdot q' = \frac{\partial}{\partial \sigma} \left\{ \frac{E[u'(W + S) \cdot W]}{q} + E[u'(W + S)] \right\}.$$

Since exponential preferences are strictly increasing and strictly prudent, we know that

$$E[u'(W + S) \cdot W] = E\{E[u'(W + S) | W] \cdot W\}$$

and

$$E[u'(W + S)] = E\{E[u'(W + S) | W]\}$$

are both increasing in σ , which implies that $q' > 0$. □

Proof of Theorem 12: Let $u(w) = -e^{-\alpha w}$, for some $\alpha > 0$. Again, $u^{[n]}(w) = (-\alpha)^n u(w)$, which implies that

$$\text{Cov}[u^{[n]}(W + q), W] = (-\alpha)^n \text{Cov}[u(W + q), W]$$

and

$$E[u^{[n]}(W + q)] = (-\alpha)^n E[u(W + q)].$$

It follows that

$$\frac{\text{Cov}[u^{[n]}(W + q), W]}{E[u^{[n]}(W + q)]} = \frac{\text{Cov}[u(W + q), W]}{E[u(W + q)]}$$

for all orders of differentiation. Theorem 11 implies that premium \hat{p} is decreasing in q , while Lemma 1 tells us that the price of the asset is increasing in σ . It then follows from Theorem 10 that condition (19), which does not hold, is necessary for \hat{p} to be non-decreasing in Σ . □

Proof of Lemma 2: We obtain this result, once again, by implicitly differentiating

$$q = u'(1) \cdot \mathbb{E}[(W + q + S)^{-\rho} \cdot (W + q)]$$

with respect to σ . By the implicit function theorem, q' equals the product of

$$\frac{u'(1)}{\mathbb{E}[1 + \rho(W + q + S)^{-(\rho+1)} \cdot (W + q) - (W + q + S)^{-\rho}]} \quad (*)$$

and

$$\mathbb{E} \left\{ \frac{\partial}{\partial \sigma} \mathbb{E}[(W + q + S)^{-\rho} \mid W] \cdot (W + q) \right\}, \quad (**)$$

so long as the denominator on the former expression is non-zero. We want that denominator to be strictly positive, which is the case since $\rho > 0$, $W + q > 0$ with probability one by assumption, and

$$\rho(w + q) > 0 \Leftrightarrow 1 + \frac{\rho(w + q)}{(w + q + s)^{\rho+1}} > \frac{1}{(w + q + s)^\rho}.$$

Since $u'(1) > 0$, it follows that the term in Eq. (*) is strictly positive.

That the term in Eq. (**) is also positive is immediate, since $(w + q + s)^{-\rho}$ is strictly convex in s , and an increase in σ is a mean-preserving spread of S given W . \square

Proof of Lemma 3: By Theorem 11, we need to argue that Eqs. (20) and (21) satisfies under the premises of the lemma. For simplicity, we divide the proof into a series of claims:

Claim 1. *Eq. (20) is satisfied*

Proof. We must argue that $u'''(W + q) \cdot \text{Var}(S \mid W)$ and W are anti-comonotone with probability one. Letting the function

$$w \mapsto u'''(w + q) \cdot \text{Var}(S \mid W = w) = \sigma \rho(\rho + 1) u'(1) w^2,$$

we have that this mapping is non-increasing so long as $w \geq 2q/\rho$. Since $q \leq \rho/2 \inf \mathcal{W}$, by assumption, we have that this inequality holds with probability 1. \square

Claim 2. *The following two inequalities, which are part of Eq. (21), also hold:*

$$\frac{\text{Cov}(U', W)}{\mathbb{E}(U')} \geq \frac{\text{Cov}(U'', W)}{\mathbb{E}(U'')}$$

and

$$\frac{\text{Cov}[U''' \cdot \text{Var}(S \mid W), W]}{\mathbb{E}[U''' \cdot \text{Var}(S \mid W)]} \geq \frac{\text{Cov}[U^{[4]} \cdot \text{Var}(S \mid W), W]}{\mathbb{E}[U^{[4]} \cdot \text{Var}(S \mid W)]},$$

Proof. We can rewrite the inequalities as

$$\frac{\mathbb{E}[(W + q)^{-\rho} \cdot W]}{\mathbb{E}[(W + q)^{-\rho}]} \geq \frac{\mathbb{E}[(W + q)^{-(\rho+1)} \cdot W]}{\mathbb{E}[(W + q)^{-(\rho+1)}]}$$

and

$$\frac{\mathbb{E}[(W + q)^{-(\rho+2)} \cdot W^3]}{\mathbb{E}[(W + q)^{-(\rho+2)} \cdot W^2]} \geq \frac{\mathbb{E}[(W + q)^{-(\rho+3)} \cdot W^3]}{\mathbb{E}[(W + q)^{-(\rho+3)} \cdot W^2]}.$$

If we define now the function

$$h(n) = \frac{\mathbb{E}[(W + q)^{-n} \cdot W^m]}{\mathbb{E}[(W + q)^{-n} \cdot W^{m-1}]}$$

over $n > 0$, given any $m \geq 0$, it suffices to show that h is non-increasing in n . By direct computation, $h'(n) \leq 0$ if, and only if,

$$\mathbb{E}[(W + q)^{-n} \cdot W^m \cdot \ln(W + q)] \cdot \mathbb{E}[(W + q)^{-n} \cdot W^{m-1}]$$

is at least as large as

$$\mathbb{E}[(W + q)^{-n} \cdot W^{m-1} \cdot \ln(W + q)] \cdot \mathbb{E}[(W + q)^{-n} \cdot W^m].$$

Letting random variable V be i.i.d. with W , this is the requirement that

$$\mathbb{E} \{ (W - V) \cdot (VW)^{m-1} \cdot [(V + q)(W + q)]^{-n} \cdot \ln(W + q) \} \geq 0.$$

This expectation is proportional, by a factor of $\Pr(V \neq W)/2$, to the sum of

$$\mathbb{E} \{ (W - V) \cdot (VW)^{m-1} \cdot [(V + q)(W + q)]^{-n} \cdot \ln(W + q) \mid V > W \}$$

and

$$\mathbb{E} \{ (W - V) \cdot (VW)^{m-1} \cdot [(V + q)(W + q)]^{-n} \cdot \ln(W + q) \mid V < W \}.$$

Since V and W follow the same distribution, the latter is

$$\mathbb{E} \{ (V - W) \cdot (VW)^{m-1} \cdot [(V + q)(W + q)]^{-n} \cdot \ln(V + q) \mid V > W \},$$

so the sum equals

$$\mathbb{E} \{ (W - V) \cdot (VW)^{m-1} \cdot [(V + q)(W + q)]^{-n} \cdot [\ln(W + q) - \ln(V + q)] \mid V > W \},$$

which is, indeed, non-negative. \square

Claim 3. *One more of the inequalities in Eq. (21) also holds:*

$$\frac{\text{Cov}[U''' \cdot \text{Var}(S \mid W), W]}{\mathbb{E}[U''' \cdot \text{Var}(S \mid W)]} \geq \frac{\text{Cov}(U'', W)}{\mathbb{E}(U'')},$$

Proof. We want to prove that

$$\frac{\mathbb{E}[(W + q)^{-(\rho+2)} \cdot W^3]}{\mathbb{E}[(W + q)^{-(\rho+2)} \cdot W^2]} \geq \frac{\mathbb{E}[(W + q)^{-(\rho+1)} \cdot W]}{\mathbb{E}[(W + q)^{-(\rho+1)}]},$$

which is equivalent to the requirement that

$$\mathbb{E}[(W + q)^{-(\rho+2)} W^3] \cdot \mathbb{E}[(W + q)^{-(\rho+1)}] \geq \mathbb{E}[(W + q)^{-(\rho+2)} W^2] \cdot \mathbb{E}[(W + q)^{-(\rho+1)} W].$$

With V defined as above, this is

$$\mathbb{E} \left\{ V \cdot [(V + q)(W + q)]^{-(\rho+1)} \cdot \left(\frac{V^2}{V + q} - \frac{W^2}{W + q} \right) \right\} \geq 0,$$

or

$$\mathbb{E} \left\{ (V - W) \cdot [(V + q)(W + q)]^{-(\rho+1)} \cdot \left(\frac{V^2}{V + q} - \frac{W^2}{W + q} \right) \mid V > W \right\} \geq 0.$$

To guarantee this, we need to argue that

$$v > w \Rightarrow \frac{v^2}{v + q} \geq \frac{w^2}{w + q},$$

or, equivalently, that the ratio $w^2/(w + q)$ is non-decreasing for $w \in \mathcal{W}$. By direct computation, this is true since $\mathcal{W} \subseteq \mathbb{R}_{++}$ and $q \geq 0$. \square

Claim 4. *The remaining inequality in Eq. (21) also holds:*

$$\frac{\text{Cov}(U', W)}{\mathbb{E}(U')} \geq \frac{\text{Cov}[U^{[4]} \cdot \text{Var}(S | W), W]}{\mathbb{E}[U^{[4]} \cdot \text{Var}(S | W)]}.$$

Proof. The desired inequality is

$$\frac{\mathbb{E}[(W + q)^{-\rho} \cdot W]}{\mathbb{E}[(W + q)^{-\rho}]} \geq \frac{\mathbb{E}[(W + q)^{-(\rho+3)} \cdot W^3]}{\mathbb{E}[(W + q)^{-(\rho+3)} \cdot W^2]},$$

which is equivalent to the requirement that

$$\mathbb{E} \left[(V - W) \cdot W^2 \cdot (V + q)^{-\rho} \cdot (W + q)^{-(\rho+3)} \right] \geq 0,$$

or, equivalently, that

$$\mathbb{E} \left\{ (V - W) \cdot [(V + q)(W + q)]^{-\rho} \cdot \left(\frac{W^2}{(W + q)^3} - \frac{V^2}{(V + q)^3} \right) \mid V > W \right\} \geq 0.$$

For this, it suffices that the ratio $w^2/(w + q)^3$ be non-increasing at all $w \in \mathcal{W}$. This is guaranteed, indeed, by the assumption that $q \geq 1/2 \inf \mathcal{W}$. \square

The four claims together yield the hypotheses of Theorem 11. \square

Proof of Theorem 13: We need to argue that

$$\frac{\mathbb{E}[(W + q)^{-\rho} \cdot W]}{\mathbb{E}[(W + q)^{-\rho}]} \leq \frac{\mathbb{E}[(W + q)^{-(\rho+2)} \cdot W^3]}{\mathbb{E}[(W + q)^{-(\rho+2)} \cdot W^2]}.$$

To see this, note again that it is equivalent to

$$\mathbb{E}[(V - W) \cdot V^2 \cdot (V + q)^{-(\rho+2)} (W + q)^{-\rho}] \geq 0,$$

or

$$\mathbb{E} \left\{ (V - W) \cdot [(V + q)(W + q)]^{-\rho} \cdot \left(\frac{V^2}{(V + q)^2} - \frac{W^2}{(W + q)^2} \right) \mid V > W \right\} \geq 0.$$

For this inequality to hold, it suffices that $w/(w + q)$ be non-increasing at all $w \in \mathcal{W}$, which is true since $\mathcal{W} \subseteq \mathbb{R}_{++}$ and $q \geq 0$. \square

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Table 1: CARA preferences: \bar{p} , p , and \hat{p}

η	α	\bar{p}	$\min\{W + S\} \div \min\{W\}$							
			95%		90%		75%		50%	
			p	\hat{p}	p	\hat{p}	p	\hat{p}	p	\hat{p}
-1	0.5	5.41%	5.42%	5.42%	5.44%	5.44%	5.56%	5.55%	5.93%	5.81%
	1	9.24%	9.26%	9.26%	9.30%	9.30%	9.54%	9.47%	10.02%	9.66%
	1.5	11.18%	11.20%	11.20%	11.24%	11.23%	11.42%	11.33%	11.70%	11.40%
	2	12.00%	12.01%	12.01%	12.03%	12.03%	12.13%	12.07%	12.26%	12.09%
	2.5	12.31%	12.32%	12.32%	12.33%	12.33%	12.38%	12.34%	12.43%	12.35%
	3	12.43%	12.43%	12.43%	12.44%	12.44%	12.46%	12.44%	12.48%	12.44%
	3.5	12.47%	12.48%	12.48%	12.48%	12.48%	12.49%	12.48%	12.49%	12.48%
	4	12.49%	12.49%	12.49%	12.49%	12.49%	12.50%	12.49%	12.50%	12.49%
0	0.5	5.41%	5.41%	5.41%	5.41%	5.41%	5.41%	5.41%	5.41%	5.41%
	1	9.24%	9.24%	9.24%	9.24%	9.24%	9.24%	9.24%	9.24%	9.24%
	1.5	11.18%	11.18%	11.18%	11.18%	11.18%	11.18%	11.18%	11.18%	11.18%
	2	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
	2.5	12.31%	12.31%	12.31%	12.31%	12.31%	12.31%	12.31%	12.31%	12.31%
	3	12.43%	12.43%	12.43%	12.43%	12.43%	12.43%	12.43%	12.43%	12.43%
	3.5	12.47%	12.47%	12.47%	12.47%	12.47%	12.47%	12.47%	12.47%	12.47%
	4	12.49%	12.49%	12.49%	12.49%	12.49%	12.49%	12.49%	12.49%	12.49%
1	0.5	5.41%	5.40%	5.40%	5.38%	5.38%	5.22%	5.24%	4.76%	4.94%
	1	9.24%	9.23%	9.23%	9.17%	9.18%	8.86%	8.97%	8.11%	8.75%
	1.5	11.18%	11.16%	11.16%	11.11%	11.12%	10.84%	11.00%	10.22%	10.92%
	2	12.00%	11.98%	11.98%	11.95%	11.96%	11.78%	11.91%	11.37%	11.88%
	2.5	12.31%	12.30%	12.31%	12.29%	12.29%	12.20%	12.27%	11.96%	12.27%
	3	12.43%	12.43%	12.43%	12.42%	12.42%	12.37%	12.42%	12.25%	12.41%
	3.5	12.47%	12.47%	12.47%	12.47%	12.47%	12.45%	12.47%	12.38%	12.47%
	4	12.49%	12.49%	12.49%	12.49%	12.49%	12.48%	12.49%	12.44%	12.49%

Table 2: CRRA preferences: \bar{p} , p , and \hat{p}

η	ρ	\bar{p}	$\min\{W + S\} \div \min\{W\}$							
			95%		90%		75%		50%	
			p	\hat{p}	p	\hat{p}	p	\hat{p}	p	\hat{p}
1	0.5	0.62%	0.62%	0.62%	0.62%	0.62%	0.63%	0.63%	0.67%	0.66%
	1	1.25%	1.25%	1.25%	1.25%	1.25%	1.27%	1.27%	1.37%	1.34%
	1.5	1.88%	1.88%	1.88%	1.88%	1.88%	1.92%	1.92%	2.10%	2.03%
	2	2.50%	2.50%	2.50%	2.51%	2.51%	2.57%	2.57%	2.86%	2.73%
	2.5	3.12%	3.12%	3.12%	3.13%	3.13%	3.22%	3.21%	3.63%	3.41%
	3	3.72%	3.72%	3.72%	3.74%	3.74%	3.86%	3.84%	4.40%	4.09%
	3.5	4.31%	4.32%	4.32%	4.34%	4.34%	4.49%	4.46%	5.16%	4.73%
	4	4.88%	4.89%	4.89%	4.91%	4.91%	5.10%	5.06%	5.90%	5.35%
2	0.5	0.62%	0.62%	0.62%	0.62%	0.62%	0.62%	0.62%	0.62%	0.62%
	1	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%	1.25%
	1.5	1.88%	1.88%	1.88%	1.88%	1.88%	1.88%	1.88%	1.88%	1.88%
	2	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%
	2.5	3.12%	3.12%	3.12%	3.12%	3.12%	3.12%	3.12%	3.12%	3.12%
	3	3.72%	3.72%	3.72%	3.72%	3.72%	3.72%	3.72%	3.72%	3.72%
	3.5	4.31%	4.31%	4.31%	4.31%	4.31%	4.31%	4.31%	4.31%	4.31%
	4	4.88%	4.88%	4.88%	4.88%	4.88%	4.88%	4.88%	4.88%	4.88%
3	0.5	0.62%	0.62%	0.62%	0.62%	0.62%	0.61%	0.61%	0.57%	0.58%
	1	1.25%	1.25%	1.25%	1.25%	1.25%	1.22%	1.22%	1.09%	1.14%
	1.5	1.88%	1.87%	1.87%	1.87%	1.87%	1.82%	1.82%	1.58%	1.69%
	2	2.50%	2.50%	2.50%	2.49%	2.49%	2.41%	2.42%	2.01%	2.23%
	2.5	3.12%	3.11%	3.11%	3.10%	3.10%	2.98%	3.00%	2.41%	2.76%
	3	3.72%	3.71%	3.71%	3.69%	3.69%	3.54%	3.57%	2.77%	3.29%
	3.5	4.31%	4.30%	4.30%	4.27%	4.28%	4.08%	4.13%	3.10%	3.81%
	4	4.88%	4.87%	4.87%	4.84%	4.84%	4.60%	4.67%	3.41%	4.33%

Table 3: Comparison with other results in the literature, *mutatis mutandis*

Reference	Model	Idiosyncratic risk	Result	Related result in this paper
Mankiw (1986), Proposition 1	Two-period economy with two equiprobable aggregate states, two idiosyncratic states, and quadratic Bernoulli function	With counter-cyclical variance	$p = \bar{p}$.	Discussion in Section 1.2
Mankiw (1986), Proposition 2	Two-period economy with two equiprobable aggregate states, two idiosyncratic states, and strictly prudent Bernoulli function	With counter-cyclical variance	$p > \bar{p}$	Theorem 1, with $\eta < 0$.
Weil (1992), Proposition 3	Two-period economy with no aggregate risk and DARA Bernoulli function for which $-u'''/u''$ is decreasing	With a-cyclical distribution	$p > \bar{p}$	Theorem 3, with $\eta = 0$.
Constantinides and Duffie (1996)	Infinite horizon economy with CRRA Bernoulli function	With counter-cyclical variance	$p > \bar{p}$. ^a	Theorem 3, with $\eta < 2$.
Storesletten et al. (2007)	OLG economy with CRRA Bernoulli function	With counter-cyclical variance	$p \approx \bar{p}$. ^b	Theorem 13, compared with Theorem 3 when $\eta = 2$.
Krueger and Lustig (2010), Proposition 3.1 and Theorem 4.2	Both two-period and infinite horizon economies with CRRA Bernoulli function	With pro-cyclical variance. ^c	$p = \bar{p}$. ^d	Theorem 3, with $\eta = 2$.

^a “If, for example, the variance of the cross-sectional distribution of consumption growth increases in economic downturns, [... a]n econometrician who does not take into account the consumer heterogeneity [...] would be overestimating the risk aversion coefficient.” (p. 230).

^b “[In comparison to the results of Constantinides and Duffie (1996), this model’s features] mitigate the ability of idiosyncratic risk to account for the observed Sharpe ratio on US equity.” (p. 519).

^c The conditional distribution of S/W is a-cyclical.

^d Furthermore, by Proposition 4.2: “the equilibrium stochastic discount factor in the Arrow and the Bond model [equals a constant times] the stochastic discount factor in the representative agent model.” (p. 23).