CURSO DE ANÁLISIS REAL PARA ECONOMÍA

Junio 15 de 2021

EXERCISE 11. *Prove that if there exists a sequence* $(x_n)_{n=1}^{\infty}$ *defined in* $X \setminus \{\bar{x}\}$ *that converges to a point* $\bar{x}$*, then* $\bar{x}$ *is a limit point of* X.

EXERCISE 12. *Consider a function* $f : X \to \mathbb{R}$*, where* $X \subseteq \mathbb{R}^K$*. Suppose that* $\bar{x} \in \mathbb{R}^K$ *is a limit point of* X *and that* $\bar{y} \in \mathbb{R}$*. Argue that* $\lim_{x \to \bar{x}} f(x) = \bar{y}$ *if for every sequence* $(x_n)_{n=1}^{\infty}$ *such that* $x_n \in X \setminus \{\bar{x}\}$*, for all* $n \in \mathbb{N}$*, and that* $\lim_{n \to \infty} x_n = \bar{x}$*, one has that* $\lim_{n \to \infty} f(x_n) = \bar{y}$.

EXERCISE 13. *Suppose that* $X = \mathbb{R}$ *and* $f : X \to \mathbb{R}$ *is defined by*

$$f(x) = \begin{cases} 1/x, & \text{if } x \neq 0; \\ 0, & \text{otherwise.} \end{cases}$$

What is $\lim_{x \to 5} f(x)$*? What is* $\lim_{x \to 0} f(x)$*?*

EXERCISE 14. *Let* $f, g : X \to \mathbb{R}$*. Let* $\bar{x}$ *be a limit point of* X*. Suppose that for numbers* $\bar{y}_1, \bar{y}_2 \in \mathbb{R}$ *one has that* $\lim_{x \to \bar{x}} f(x) = \bar{y}_1$ *and* $\lim_{x \to \bar{x}} g(x) = \bar{y}_2$*. Argue that*

$$\lim_{x \to \bar{x}} (f + g)(x) = \bar{y}_1 + \bar{y}_2.$$