Mathematical Social Sciences 121 (2023) 8–19

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

Insurance contracts and financial markets

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ARTICLE INFO

Article history:
Received 4 February 2022
Received in revised form 10 August 2022
Accepted 15 November 2022
Available online 26 November 2022

Keywords:
Insurance
Screening
Pooling
Finance

ABSTRACT

We study the interaction between insurance and financial markets. Individuals who differ only in risk have access to insurance contracts offered by a monopolist and can also save through a competitive market. We show that an equilibrium always exists in that economy and identify an externality imposed on the insurer's decision by the endogeneity of prices in the financial market. We argue that, because of that externality and in contrast to the case of pure contract theory, equilibrium always exhibits under-insurance even for the riskiest agents in the economy and may even exhibit pooling. Importantly, the externality does not disrupt the single crossing property of the economy.

Published by Elsevier B.V.

1. Introduction

In this paper, we couple a version of the standard monopolistic insurance model to a competitive financial market. In an economy that presents idiosyncratic risk, a monopolist charges a premium in the present in exchange for idiosyncratic state-contingent promises of coverage in the future. At the same time, a financial market that is complete with respect to aggregate shocks is available to the agents. Individuals with different risk distributions simultaneously choose a portfolio of financial assets and whether to buy an insurance contract, and which. At equilibrium, these choices must be individually rational, and asset prices must guarantee that the financial markets clear. The monopolist understands this, and optimally chooses a menu of insurance contracts that is self-enforcing, taking into account how this menu impacts the equilibrium of the financial market and, therefore, the insurees' willingness to pay for insurance.

The motivation for this study is that abstracting away from any other relation or trade occurring in the economy may sometimes render the classical insurance theory analysis invalid. The considerations that explain the behavior of, say, the insurance market for domestic appliances would likely be insufficient to understand the effects of a social security reform.[1] In particular, when the insurance industry is considered in isolation, its relation with other financial markets is overlooked: on one hand, the analysis will ignore that the distribution of wealth in the society is endogenous, as individuals react to different insurance coverages by purchasing different financial portfolios; on the other, it will ignore that the prices of financial assets affect consumers' willingness to pay for insurance.

Our main result is that the equilibrium menu of contracts always displays under-insurance of *all* agents, even the riskiest ones, which differs from the usual lessons obtained in partial equilibrium. Moreover, the equilibrium menu can pool agents of different riskiness together, which is in stark contrast with the basic results of contract theory.

We then identify the driving force behind these results, which we dub the *market effect*. Suppose that the firm decides to increase the coverage for any insuree type. This has two effects. One is direct: the contract that insuree signs becomes more attractive. But under general equilibrium, there is also an indirect effect: when that type of agent receives more coverage, their need for savings decreases and, consequently, so does the price of savings. This price drop affects *all* the agents in the economy, making the financial assets relatively more attractive and potentially making some agents opt-out of insurance altogether. In other words, the change in coverage for one type of insuree affects the participation decision of all others. Because the insurance company understands this mechanism, the optimal menu takes the market effect into account.

The market effect is conceptually different from other sources of inefficiency and pooling in the contracting literature. It consists of an externality that operates through the individual rationality constraint, rather than through incentives. It makes separation costly for the monopolist by making it more expensive to ensure the participation of individuals in a potential separating menu. This conclusion differentiates our result from the literature that

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1 And the latter can be highly significant: see Brevoort et al. (2017).

https://doi.org/10.1016/j.mathsocsci.2022.11.002
0165-4896/Published by Elsevier B.V.

deals with the failure of the Spence–Mirrlees condition in mechanism design. Additionally, our result is not a consequence of countervailing incentives and type-dependent reservation utilities either. Rather, in our model the outside options do depend on private information, on the specific contract each agent is offered and, what is new, on the whole set of contracts posted by the firm. In fact, the externality is generated by the endogeneity of all agents' reservation utilities to the whole menu of contracts, which is in itself a consequence of individual decisions being linked together by the financial market.

2. Related literature

Our results are meant to be a contribution to the third-best literature when contracts are not exclusive. Jaynes (1978) was first to observe that the implications of Rothschild and Stiglitz (1976), Stiglitz (1977) and Wilson (1977) are not robust to settings in which the insurer is unable to monitor all the transactions of the insurees. Later, Allen (1985) observed that in a repeated principal–agent problem where the intertemporal decisions of the agent can be used to discern information about her type, the conclusions of the one-shot version of the model are likely to fail, while Hammond (1987) showed that when there is unobservable outside trade, there are very strong limits to redistribution, and the ability of contracts to reallocate risks in an economy gets severely restricted. Indeed, Netzer and Scheuer (2009) recognized that trade in related markets may induce failures of the Spence–Mirrlees property that underlies the properties of separating equilibria. But except for some works in public finance,[2] mechanism design problems have been solved in a partial equilibrium approach. The insights of those analyses operate through the incentive compatibility constraints and are different from ours. In general, our point is that, to the extent that general equilibrium effects are relevant, participation constraints may play as important a role as the one of incentives, through the endogeneity of the outside options.

In terms of topic, the paper studies the inter-relation between contract economics and general equilibrium. A wide literature has been concerned with this issue since (Prescott and Townsend, 1984): see Dubey and Geanakoplos (2002) and Rustichini and Siconolfi (2008), or, from a different perspective, Bisin et al. (2011). These papers generally study economies that are competitive and subject to informational problems that generate the possibility of adverse selection. In these economies, they model contracts as exclusive relations between a given firm and a consumer. The contracts, which are usually lotteries on future consumption, are either offered by firms that are competitive or are exogenously given. The latter is the case, for instance, in Dubey and Geanakoplos (2002), where individuals buy insurance by choosing among a number of pools to trade in. Contracts are sold in a competitive market in which both firms and agents take prices as given and, in equilibrium, prices reflect the profile of types of agents buying them.

This is in contrast with our model, in which the insurer is a monopolist company exercising power over not only the price of its contracts, as in Stiglitz (1977) and Chade and Schlee (2012), but also over other prices. The key aspect here is that we assume a market structure that is divided in two. On one hand there is the contract market, which is strategic, and simultaneously, on the other hand there is a financial market that is competitive.

Bisin and Gottardi (2006) follows a path similar to ours, by identifying the externality that is associated to adverse selection in the competitive economy. In their model, competitive firms sell promises of future consumption in separated markets, one for each type of agent (who vary only in risk). Risk is private information but, in equilibrium, insurees self-select to the market designed to their own type. Exclusivity of contracts and the fact that markets are competitive and naturally separated precludes risk-sharing between types of agents and generates inefficiency, and it turns out that the level of net trades an agent can make in a market is linked to trades in the other one through incentive compatibility.

We, too, identify an externality that the existence of competitive markets imposes on the screening problem. But, in contrast, our externality is reflected directly in the problem of the firm, rather than on another individual's problem. This is so because we assume a firm that internalizes the possible deviations by agents. If we see this work as trying to understand contract theory in the realm of general equilibrium, it inverts what is usually done in the mentioned literature: we take competitive markets as the background, and look at the problem in a mechanism design framework.

Another related paper is Farhi et al. (2009), which, as anticipated by Hammond (1987), observes the inefficiency of competitive equilibria when agents can engage in multilateral, unobservable trades. These hidden trades become an externality that agents fail to internalize, so if a planner tries to allocate more resources to agents with high marginal valuations, agents with lower valuation absorb some of those resources and limit the effect of the planner's policy. Importantly, it is again the failure of self-selection mechanisms that induce the inefficiency, unlike in our case where the externality realizes through a participation constraint.

Our results complement those from Attar et al. (2011), where a seller faces a group of buyers who compete by offering the seller non-exclusive contracts, and from Ales and Maziero (2016), where multiple insurers compete with non-exclusive contracts. The substantive difference is that in these papers the distortions arise through the incentive compatibility constraints, while our market effect operates through the participation constraints.[3] In that sense, closer insights come from Peters and Severinov (1997) and Virág (2010), for the case of competition between different auctioneers. There, the decisions of a seller affect the reservation price that other seller may charge to continue to attract potential buyers optimally.

Lastly, our paper provides a new justification for pooling that is not solely due to either of the two most usual ones: the absence of Spence–Mirrlees conditions as in, for instance, Araujo and Moreira (2010); or countervailing incentives as in Maggi and Rodriguez-Clare (1995).[4] In our model, we show that incentive constraints do not pose a problem for separation: a condition that plays the role of the single-crossing property is preserved by the endogenous counterparts of utility functions in our framework. As for the latter, countervailing incentives in the sense of Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995), and Jullien (2000), are indeed present in our work. Because in our model

[2] See, for instance, Golosov and Tsyvinski (2007) and da Costa (2009). The focus of these papers, optimal taxation, is different from ours.

[3] Also related, though in a setting of moral hazard, is Bisin and Guaitoli (2004).

[4] Countervailing incentives happen when a given agent may have incentives either to understate or overstate their private information depending on the actual realization of such private variable. In many cases that is associated with type-dependent outside options, usually when trading with the firm demands the agent to forego an opportunity outside the contractual relation that varies with his type. In pure contract theoretical models of insurance, foregone opportunities are always present, but type-dependence of the outside options is easily controlled as incentive compatibility implies that the difference between the inside outcome and the outside option is monotone in types.

agents can use the financial market to alter their own levels of wealth, countervailing incentives naturally arise and, moreover, they do so endogenously. Therefore, we are adding an extra layer of mixed incentives to the countervailing phenomenon. In our model, outside options are not only type-dependent, but also endogenous because two different contracts generate two levels of savings that affect distinctly the relation between the insurance outcome and the outside option. However, the key factor behind pooling in the economies we study is not that individuals endogenously react to the offer of contracts — that also happens, for instance, in Netzer and Scheuer (2009) . Rather, the driving force for pooling here is that the effect of such reaction leads to changes that affect all the constraints in the economy, including other agents' participation constraints. Namely, individual deviations, when occurring for all individuals of a given type, alter the prices in the competitive financial market and, therefore, have a widespread effect in the whole feasibility set, and not only in their own constraints.

3. The environment

We present next the simplest setting that captures our main insights: a two-period version of the canonical monopolistic insurance problem posited by Stiglitz (1977) and fully developed by Chade and Schlee (2012). We assume that the economy evolves over the present and a future period, and consists of only one commodity per period.

3.1. The population and their risks

The population is a continuum of individuals whose mass we normalize to one. In the future, independently, each individual will find herself in one of two idiosyncratic states: she may have an accident, or not. We denote these idiosyncratic states, respectively, by A and N. If an individual does not suffer the accident, her wealth is $\bar{\omega} > 0$; if she does, she sustains a loss of $0 < \lambda < \bar{\omega}$, which results in a wealth of $\bar{\omega} - \lambda$.

There are two types of agent, L and H. The only difference between the two types is the probability with which they have the accident, with individuals of type H being more likely to have it: we denote by π^i the probability that a person of type i experiences the accident in the future, and assume that $\pi^H > \pi^L$. The fraction of type-i individuals in the population is μ^i.

If an individual of type i has a consumption plan x, consisting of x_0 units in the present, x_A units upon having the accident in the future, and x_N units otherwise, her *ex-ante* utility is

$$U^i(x) = x_0 + \pi^i u(x_A) + (1 - \pi^i)u(x_N). \tag{1}$$

The cardinal utility index $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly risk averse, strictly prudent and differentiable three times on $\mathbb{R}_{++}$, and satisfies the standard Inada conditions.[5]

We do not impose any bounds on present consumption so, having assumed that preferences are quasi-linear in that variable, individual endowments for that period play no role in our analysis.[6]

3.2. The insurance firm

An insurance contract specifies the premium charged in the present and the amount that is paid to the insuree if she has the accident in the future. Denote these terms by p and a, respectively. In the absence of other trades, upon signing an insurance contract the agent has a present wealth of $-p$; in the future, her wealth is $\bar{\omega}$ if she does not have the accident, and $\bar{\omega} - \lambda + a$ if she does. It will be useful to define $w = \bar{\omega} - \lambda + a$ and to write the insurance contracts as pairs (p, w). We assume that $a \in [0, \lambda]$, so that $w \in [\bar{\omega} - \lambda, \bar{\omega}]$, and refer to w as the *coverage* of the contract.

There is a monopolistic insurance company in the economy. Its goal is to maximize its present profit. In that goal, the firm is subject only to the constraint that it must constitute the portfolio that allows it to honor its future coverage commitments. In the first period, this company posts a menu of contracts, M, from which each agent selects at most one. We consider only menus of the form $M = \{(p^H, w^H), (p^L, w^L)\}$, where there is one contract that is recommended for each type. This implies no loss of generality.

Given the choices of the individuals, the insurance company collects the resulting premia as revenue. In order to be able to honor these contracts, the firm must constitute the savings needed to cover its *ex-post* obligations. By the law of large numbers, the aggregate obligation of the firm is riskless: assuming that contract (p^i, w^i) attracts the individuals of type i, and only them, the firm must guarantee future funds of $\mu^i \pi^i [w^i - (\bar{\omega} - \lambda)]$ in order to cover the obligations acquired with this type of insuree.

Both the revenue collected and the savings required are determined by the menu offered by the firm. Denoting these variables as R and Y, respectively, the monopolist's profits are

$$G(M) = R(M) - q(M) \cdot Y(M), \tag{2}$$

where $q(M)$ is the price of a riskless bond. By definition, $R(M) = \mu^L p^L + \mu^H p^H$ and

$$Y(M) = \mu^L \pi^L [w^L - (\bar{\omega} - \lambda)] + \mu^H \pi^H [w^H - (\bar{\omega} - \lambda)]. \tag{3}$$

Importantly, for Eq. (2) to be a valid objective function for the firm, it must consider only menus where the insurees self-select to the contract recommended for their respective types and must involve a correct understanding of how the choice of the menu affects the price of the bond.

3.3. The financial market

Besides buying insurance contracts, all individuals can competitively demand a riskless bond. This instrument allows them to transfer wealth across time periods, and is therefore an imperfect substitute for insurance. The holdings of an individual of type i are denoted as y^i.

Recalling that Y is the demand of the insurance company, the financial market clears when

$$\mu^L y^L + \mu^H y^H + Y = 0. \tag{MC}$$

To be sure, note that we assume that the insurance and financial markets operate simultaneously. The earlier literature – Jaynes (1978) and Hammond (1987) – already made the point that in a setting with multiple insurers, there would be incentives for keeping some trades hidden from other agents. As Attar et al. (2011) pointed out more recently, "competition on financial markets is often nonexclusive, as each agent can trade with multiple partners who cannot monitor each others' trades with the agent". These ideas continue to motivate our paper: modern financial markets provide many instruments with which agents can trade "unobservably". Most clearly, the so called "dark pool trading" has appeared with this explicit purpose. Or, with the wide availability of derivatives, it is possible to constitute a portfolio of assets that undoes an agent's declared financial position, even if short-selling is forbidden.

5 That is: $u' > 0$, $u'' < 0$, $u''' > 0$, $\lim_{x \downarrow 0} u'(x) = \infty$ and $\lim_{x \to \infty} u'(x) = 0$.

6 See Program (ISP) below.

4. Equilibrium

4.1. Individual choice

If an individual of type i has taken an insurance contract (p, w) and the price of the bond is q, her optimal level of savings solves the program[7]

$$\max_{y} \left\{ -p - qy + \pi^i u(w + y) + (1 - \pi^i)u(\bar{\omega} + y) : y \geq -w \right\}. \tag{ISP}$$

Let $V^i(p, w, q)$ represent the resulting indirect utility function. The individual's choice of an insurance contract is, then, the solution of the program

$$\max_{(p,w)} \left\{ V^i(p, w, q) : (p, w) \in M \cup \{(0, \bar{\omega} - \lambda)\} \right\}. \tag{IIP}$$

For simplicity of notation, we will write $\bar{M} = M \cup \{(0, \bar{\omega} - \lambda)\}$. This set contains the menu of contracts offered by the monopolist, as well as the no-insurance option that is always available to the individual. With this notation, we can re-write Program (IIP) as stating that the agent chooses a $(p, w) \in \bar{M}$ such that,

$$V^i(p, w, q) \geq V^i(\tilde{p}, \tilde{w}, q) \text{ for all } (\tilde{p}, \tilde{w}) \in \bar{M}. \tag{IC and IR}$$

Of course, the individual chooses her insurance contract and her savings simultaneously. The nested manner in which we wrote the problem is equivalent to the simultaneous choice and will be useful for the purposes of presentation. To be sure, note that in the choice of a contract the individual takes into account the way in which that choice will affect her level of savings. Also, note that Program (IIP) summarizes the two canonical considerations of contract theory: the incentive compatibility (IC) constraint, and the individual rationality (IR) constraint.

4.2. Financial market equilibrium

Definition 1. Given a menu M, an equilibrium in the financial market, FME, is a triple $\{y^H, y^L, q\}$, consisting of a level of savings for each type and a price for the bond, such that:

1. for each i, y^i solves Problem (ISP), given q and (p^i, w^i);
2. for each i, (p^i, w^i) solves Problem (IIP), given q and M; and
3. Eq. (MC) holds with $Y = Y(M)$, so the bond market clears.

This concept serves as a prediction, from the perspective of the insurance company, of the general equilibrium implications of the firm's choice of a menu.[8] Assuming that each insuree chooses the contract intended for them, the first and third conditions determine the equilibrium demands and price in the market for the riskless bond. Using the indirect utility functions at equilibrium prices, the second condition says that all individuals choose to participate in the insurance market and self-select to their contracts, as the firm intended.

4.3. Equilibrium

Denote by $E(M)$ the set of FME for menu M:

Definition 2. An *equilibrium* is a tuple $\{M^*, y^H, y^L, q\}$, consisting of a menu of contracts, individual savings for each type and a price for the bond, such that:

1. given M^*, individual savings and the asset price constitute an FME: $\{y^H, y^L, q\} \in E(M^*)$, and
2. menu M^* solves the firm's maximization problem:

$$\max_{M} \{G(M) : E(M) \neq \varnothing\}. \tag{FMP}$$

Again, the first condition states that the insurance firm understands that the menu of contracts affects the financial market, and that its predictions are consistent with that effect. The second says that, given that understanding and the cost of transferring funds to the future, the firm chooses the optimal menu. In this condition, problem (FMP) guarantees that the firm is ruling out menus for which, at equilibrium prices, either an IR or an IC constraint would fail.[9]

5. Existence of equilibrium

Note that an equilibrium exists if and only if the firm's maximization problem has a solution. In order to show that that is the case, we first use the quasi-linear structure of the economy to write the non-emptiness constraint in Program (FMP) in a more tractable way. Then, our argument shows that the firm does not have unlimited market power, in the sense that it cannot construct menus giving itself unbounded profits and which are still acceptable to the agents. This is so because the firm's capacity of manipulating outside options is naturally constrained by agents' ability to use the market as a substitute for expensive insurance.

Proposition 1. *An equilibrium exists.*

Proof. Denote by $y^i(w, q)$ the solution of Program (ISP), noting that it does not depend on p, by quasi-linearity. Denote also by $Q(w^L, w^H)$, the set of all q such that

$$\mu^L\{y^L(w^L, q) + \pi^L[w^L - (\bar{\omega} - \lambda)]\} \\ + \mu^H\{y^H(w^H, q) + \pi^H[w^H - (\bar{\omega} - \lambda)]\} = 0.$$

It is immediate, from the first-order conditions of (ISP), that $q \in Q(w^L, w^H)$ if, and only if,

$$q = \pi^L u'(w^L + y^L) + (1 - \pi^L)u'(\bar{\omega} + y^L)$$

for some $y = (y^L, y^H)$ that maximizes

$$\sum_i \mu^i \left[\pi^i u(w^i + \tilde{y}^i) + (1 - \pi^i)u(\bar{\omega} + \tilde{y}^i)\right] \tag{4}$$

subject to the constraints that

$$w^i + y^i \geq 0 \text{ and } \sum_i \mu^i\{\tilde{y}^i + \pi^i[w^i - (\bar{\omega} - \lambda)]\} = 0. \tag{5}$$

By continuity, this program has a solution, which is unique by strict concavity. It follows that set $Q(w^L, w^H)$ is a singleton, so we can denote its unique element by $q(w^L, w^H)$.

[7] Strictly speaking, present consumption should be $x_0 = \omega_0 - p - qy$, where ω_0 would represent date-0 wealth. Since we are not requiring $x_0 \geq 0$, ω_0 is a constant in this problem and we can just ignore it.

[8] This is an important feature of our model: the insurance monopolist does understand the "general equilibrium" effects of the choice of a menu. While the assumption of a monopolist has been common in the insurance literature, in this paper it also implies this "prediction sophistication", which was not present in the earlier literature.

[9] A subtle potential difficulty is the possibility that set $E(M)$ be non-singleton: in the presence of multiple financial equilibria, we would need to stipulate a mechanism by which the firm selects one. Since we are considering preferences that are quasilinear in present consumption, the literature on general equilibrium guarantees that the set of arrays that satisfy the first and third conditions of the definition of FME is a singleton. Adding the second condition of that definition gives us that $E(M)$ is either empty or a singleton, so we do not need to worry about multiplicity of FME.

With this notation, we can re-write Program (FMP) as

$$\max_M \left\{ G(M) : \forall (p,w) \in \bar{M}, \right.$$
$$\left. V^i(p^i, w^i, q(w^L, w^H)) \geq V^i(p, w, q(w^L, w^H)) \right\}, \qquad \text{(FMP')}$$

and it suffices to show that this program has a solution. We first observe that its domain is compact. By Berge's theorem, continuity of the indirect utility functions guarantees that the domain is closed. Coverages are bounded, by assumption. As for premia, it suffices to consider the individual rationality constraints. For each individual, the surplus gained by taking the intended insurance contract, when the coverages offered are $w = (w^L, w^H)$ and insurance is free, is at most

$$\max_i \left\{ \max_w \left\{ V^i(0, w^i, q(w)) - V^i(0, \bar{\omega} - \lambda, q(w)) : \right.\right.$$
$$\left.\left. w \in [\bar{\omega} - \lambda, \bar{\omega}]^2 \right\} \right\}$$

which exists by continuity and compactness. This value is an upper bound for the premia that the insurance company can charge, if it is to satisfy that $V^i(p^i, w^i, q(w)) \geq V^i(0, \bar{\omega} - \lambda, q(w))$.

To complete the proof, we just need to argue that the objective function of Program (FMP'), namely the function G defined in Eq. (2) is continuous. That the functions R and Y are continuous is straightforward. The following lemma shows so is function q. $\square$

The demand and price functions defined in the previous proof are well understood in general equilibrium theory. Some of their comparative statics are introduced in the following lemma and proven in the Appendix.

Lemma 1. *Both functions y^i and function q are continuously differentiable and strictly decreasing.*

Program (FMP') is a characterization of the problem of the firm that resembles the problem of mechanism design in partial equilibrium environments. The challenge is that (FMP') is written in terms of indirect utility functions evaluated at market-clearing prices, so any perturbation in the menu affects the whole set of constraints. That complete dependence of the constraint set on all the entries of the menu is precisely the consequence of relaxing the assumption of an isolated insurance market.[10] The rest of the paper is devoted to our two main results: that when insurees can save in a financial market even the highest risk agent is underinsured and that types can be pooled in an equilibrium.

6. Distortions also at the top

Our concept of equilibrium involves three deviations from the standard Arrow–Debreu model. First, the financial market is incomplete in the sense that the agents cannot trade assets whose payoff is contingent in the occurrence of the accident. Instead, the agents can buy insurance contracts, but the insurance company has market power, which is in itself a second market failure. And third, obviously information is asymmetric.

The goal of this section is to understand the distortions that arise in equilibrium. Based on the contract theory literature, our emphasis is to determine whether there are "distortions at the top": do the riskier agents get their "efficient" contract? In order to answer this question, we perform three exercises:

1. We consider the insurance problem in the absence of the financial market, and confirm that the usual prescriptions of contract theory are valid in this case.
2. We introduce the financial market but with a simplification: we assume that the insurance company constitutes its portfolio at an exogenously given price. The motivation for this exercise is to turn off the second inefficiency mentioned above: the insurer will not exercise any market power over the price at which it saves.
3. Finally, we consider the full equilibrium concept where the insurer pays the endogenously determined price for the riskless savings

The main insight of these three results is that in the presence of concurrent financial markets, the insurer finds it optimal to under-insure even the riskier agents: their marginal utility from the coverage offered by the firm is higher than the firm's marginal cost of increasing such coverage.

6.1. Benchmark: the economy without financial markets

In order to have a background against which to compare, we now characterize equilibrium in the case where the insurees cannot save. In such setting, the value of contract (p, w) for an agent of type i would be simply

$$\tilde{U}^i(p, w) = -p + \pi^i u(w) + (1 - \pi^i) u(\bar{\omega}),$$

while the insurer's problem would be

$$\max_M \left\{ G(M, \kappa) : \forall (p, w) \in \bar{M},\ \tilde{U}^i(p^i, w^i) \geq \tilde{U}^i(p, w) \right\},$$

where κ denotes the price at which the insurer can buy a riskless bond, and

$$G(M, \kappa) = R(M) - \kappa \cdot Y(M). \tag{6}$$

In order to concentrate only in the most interesting case, we assume that

$$u'(\bar{\omega}) < \kappa < u'(\bar{\omega} - \lambda). \tag{7}$$

The following proposition makes it clear that our two-period version of the insurance problem is but an instance of the canonical monopolistic insurance model of Stiglitz (1977).

Lemma 2. *In the absence of financial markets, the optimal menu is such that $U^L(p^L, w^L) = U^L(0, \bar{\omega} - \lambda)$, $U^H(p^H, w^H) = U^H(p^L, w^L)$, and $u'(w^H) = \kappa$ and $u'(w^L) > \kappa$.*

The first two claims of the lemma say that the safest type of agent is left indifferent to her outside option, while the riskiest type is indifferent between the two contracts in the menu. Th third one says that the type-H contract is undistorted in comparison to the Pareto optimal allocation, by receiving insurance so that $u'(w^H) = \kappa$, while the contract for type L individuals is distorted. It follows that separation and the absence of distortions "at the top" are properties of equilibrium in the absence of a financial market.

6.2. External funding for the insurer

Consider now the economy with a financial market, but suppose that the insurance company *cannot* trade in the same market as the insurees. Instead, it can buy riskless bonds in an external (unmodeled) market at a fixed price of κ.[11]

10 Quasi-linearity introduces a partial separation between contracts and general equilibrium: the price of savings does not depend on the premia. This simplicity gives us tractability but does not amount to making the problem one of partial equilibrium. In fact, the price of savings still depends on the coverages of the insurance contracts.

11 This model may be of interest by itself. Fortunately, the existence result of Proposition 1 and the comparative statics of Lemma 1 extend readily to this case, and it is not necessary to assume that the agents are prudent.

Only two changes are necessary to study this case. First, the profits of the insurance company become

$$G(M) = R(M) - \kappa \cdot Y(M), \tag{8}$$

instead of Eq. (2). Second, the market clearing requirement is simply that

$$\mu^L y^L + \mu^H y^H = 0,$$

instead of Condition (MC). Subject to these two changes, the definition of equilibrium is still given by Definitions 1 and 2. For the sake of clarity, we refer to this case as *equilibrium with external funding.*[12]

Our first result is that, unlike in the model without financial markets, at equilibrium there are distortions from the first best even for the riskier type of insuree.

Proposition 2. *In the economy with financial markets, let $\{M, y^H, y^L, q\}$ be an equilibrium with external funding. If the allocation is interior, in the sense that $\bar{\omega} > w^H \geq w^L > \bar{\omega} - \lambda$, then*

$$u'(w^L + y^L) > u'(w^H + y^H) > \kappa.$$

Proof. The individual rationality constraint for individuals of type H is redundant, so only their incentive compatibility constraint binds in the firm's maximization problem, along with the individual rationality constraint of the type L contract. Using interiority, the first-order conditions of the firm's problem with respect to p^H and w^H are that

$$\mu^H = -\ell^H \tfrac{\partial V^H}{\partial p^H}(p^H, w^H, q) \tag{9}$$

and

$$\kappa \mu^H \pi^H = \ell^H \tfrac{\partial V^H}{\partial w^H}(p^H, w^H, q) + \tfrac{\partial q}{\partial w^H}(w) \cdot D, \tag{10}$$

where ℓ^H and ℓ^L are, respectively, the multipliers of the incentive compatibility constraint of type H and the individual rationality constraint of type L, and

$$\begin{aligned} D = {} & \ell^L \left[\tfrac{\partial V^L}{\partial q}(p^L, w^L, q) - \tfrac{\partial V^L}{\partial q}(0, \bar{\omega} - \lambda, q) \right] \\ & + \ell^H \left[\tfrac{\partial V^H}{\partial q}(p^H, w^H, q) - \tfrac{\partial V^H}{\partial q}(p^L, w^L, q) \right]. \end{aligned} \tag{11}$$

By the envelope theorem, the Eqs. (9) and (10) above can be rewritten as

$$\mu^H \pi^H (u'(w^H + y^H) - k) = -\tfrac{\partial q}{\partial w^H}(w) \cdot D. \tag{12}$$

Lemma 1 tells us that $\partial q / \partial w^H < 0$, while, using Roy's identity,

$$\begin{aligned} D = {} & \ell^L [y^L(0, \bar{\omega} - \lambda, q) - y^L(p^L, w^L, q)] \\ & + \ell^H [y^H(p^L, w^L, q) - y^H(p^H, w^H, q)]. \end{aligned}$$

Because incentive compatibility requires $w^H \geq w^L$, by Lemma 1 we have that $D > 0$ and the result follows from Eq. (12). □

The proof of Proposition 2 reveals the main difference between this model and one without financial markets. Suppose that the insurer considered increasing the coverage for the H type marginally. In the model with no financial markets, the insurer does not need to provide extra incentives for the L type, so this increase in coverage only affects the total surplus. Because of that, the firm chooses w^H so as to maximize the surplus, i.e. $u'(w^H) = k$. In the presence of financial markets, however, Eq. (12) shows that this same marginal increase in the coverage of the H type has an indirect effect. By increasing their coverage, the insurer is effectively reducing the demand for savings in the financial market, thereby decreasing the price of savings. This decrease affects the outside option of both agents and, in particular, makes the individual rationality constraint harder to satisfy for the L type. Therefore, when considering the same increase in coverage, the seller needs to take into account their added cost of ensuring participation.

6.3. Internal funding for the insurer

The next proposition says that, after taking into account the market power of the insurance company, in the case of internal funding the equilibrium allocation also displays distortions at the top: the marginal utility of the riskier types in the accident state is above the marginal cost that the firm incurs by improving that type's coverage.

Proposition 3. *In the economy with financial markets and internal funding, let $\{M, y^H, y^L, q\}$ be an equilibrium. If the allocation is interior, then*

$$u'(w^H + y^H) > q + \frac{1}{\mu^H \pi^H} \frac{\partial q}{\partial w^H} \cdot Y(M).$$

Proof. The argument is similar to the previous proof. The first-order conditions with respect to p^H and w^H continue to be Eqs. (9) and (10), except that the term in Eq. (11) is augmented to

$$\begin{aligned} D = {} & \ell^L \left[\tfrac{\partial V^L}{\partial q}(p^L, w^L, q) - \tfrac{\partial V^L}{\partial q}(0, \bar{\omega} - \lambda, q) \right] \\ & + \ell^H \left[\tfrac{\partial V^H}{\partial q}(p^H, w^H, q) - \tfrac{\partial V^H}{\partial q}(p^L, w^L, q) \right] - Y(M). \end{aligned}$$

Again, using the envelope theorem, the two equations above can be rewritten as

$$\mu^H \pi^H [u'(w^H + y^H) - q) = -\tfrac{\partial q}{\partial w^H}(w) \cdot D,$$

and Lemma 1 tells us that $\partial q / \partial w^H < 0$. As before, using Roy's identity,

$$\begin{aligned} D = {} & \ell^L [y^L(0, \bar{\omega} - \lambda, q) - y^L(p^L, w^L, q)] \\ & + \ell^H [y^H(p^L, w^L, q) - y^H(p^H, w^H, q)] - Y(M). \end{aligned}$$

Since $w^H \geq w^L > \bar{\omega} - \lambda$, by Lemma 1 we get that $D > -Y(M)$. □

7. Pooling at equilibrium

Our next result is that separation is *not* a robust property of the equilibrium contracts in our setting with endogenous savings.[13] Pooling occurs if the market effect induced by the endogeneity of the outside options is strong enough to dominate the incentives to separate, which come from the ordering property. In this section we argue that, indeed, the market effect may dominate:

> *In the presence of a financial market, there exist economies where the two types are offered the same insurance contract at equilibrium.*

7.1. External funding for the insurer

For simplicity of presentation, we first develop the result for equilibria with external funding. In order to see economies where a pooling menu arises at equilibrium, assume that the individuals have CARA cardinal utility indexes with absolute risk aversion $\alpha > 0$; that is, let $u(x) = -e^{-\alpha x}$.

[12] To avoid confusion, when needed we refer to the original definition as *equilibrium with internal funding.*

[13] Recall that we distinguish pooling from market exclusion: exclusion happens when any of the individuals is left receiving zero insurance; pooling, when both individuals receive the same non-zero contract.

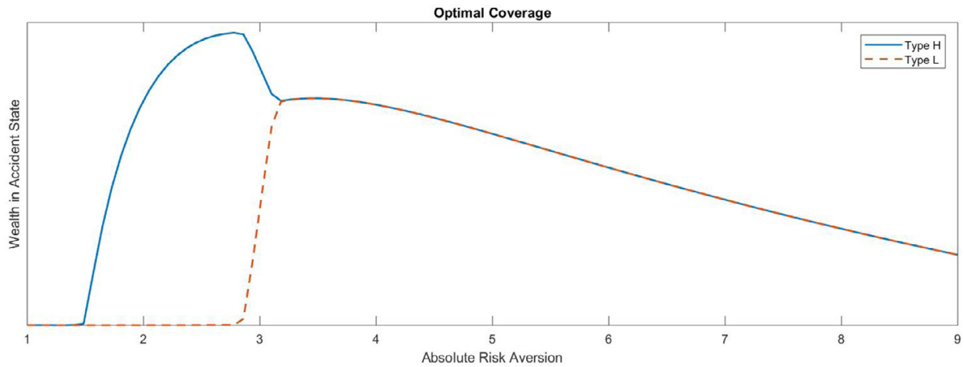

Fig. 1. Equilibrium values of w^H and w^L.

We proceed by first solving for FME in closed form, which provides us with the functions needed for the monopolist's problem. First, suppose that an i-type individual accepts the contract (p^i, w^i). Then, his optimization problem at savings price q is:

$$\max_{y \in \mathbb{R}} \left\{ -p^i - yq - F^i(w^i) e^{-\alpha y} \right\},$$

where $F^i(x) = \pi^i e^{-\alpha x} + (1 - \pi^i) e^{-\alpha \bar{\omega}}$. The first-order conditions of this problem, which are necessary and sufficient, can be rearranged and rewritten in logarithmic form so as to get

$$y^i = \frac{1}{\alpha} \ln \frac{\alpha}{q} F^i(w^i).$$

Using market clearing, individual savings vanish and we can find the equilibrium price as a function of the two insurance coverages. Defining $f(x, y) = F^H(x)^{\mu^H} F^L(y)^{\mu^L}$, we have $q = \alpha f(w^H, w^L)$.

As a consequence, it is possible to characterize savings for an agent of type i when accepting coverage w, if the profile of coverages in society is (w^H, w^L), as

$$y^i(w, q(w^H, w^L)) = \frac{1}{\alpha} \ln \frac{F^i(w)}{f(w^H, w^L)}.$$

And, finally, indirect utilities at the FME are

$$V^i(p, w, q(w^H, w^L)) = -p - f(w^H, w^L) \left[\ln \frac{F^i(w)}{f(w^H, w^L)} - 1 \right].$$

Noticing that terms that are constant between types cancel out, we can replace the indirect utility functions by the simpler functions

$$v^i(p, w, q(w^H, w^L)) = -p - f(w^H, w^L) \ln F^i(w).$$

This value function is the composition of the indirect utility function and the equilibrium price function. The expression is informative in that it separates precisely the two effects our model is interested in. That is: on one hand, there is the direct effect of coverage on utility, measured by $-\ln F^i(w)$; on the other, changes in coverage for any type have an indirect effect via market changes, and the $f(w^H, w^L)$ term captures that.

We can now solve the firm's problem numerically. Because the problem lacks concavity, we cannot rely directly on standard local optimization algorithms and need to resort to `grid-optmization` and `multistart` methods that provide, respectively, approximations of the global optimum and robustness checks.

Table 1 shows the parameter specification for which the optimal menus were calculated. The risk aversion parameter, α, is not present in the table because we let it vary. We want to see whether the optimal menu of contracts presents pooling or separation.

Table 1
Parameter specification.

$\bar{\omega}$	λ	μ^L	π^H	π^L	κ
1	0.8	0.3	0.6	0.3	1

Fig. 1 shows the values of w^H and w^L at equilibrium, for different values of α. The graph shows that for very small risk aversion insurance is absent in this economy. This is compatible with Lemma 2, for the case without financial markets. As α increases, coverage is first improved for the riskier type but, when individuals get more risk averse, pooling appears.

7.2. Internal funding for the insurance company

Consider now the case of internal funding, and suppose that the firm is considering whether a separating deviation from a given pooling contract will improve its profits. Our argument above that the latter need not be the case, since better insured high-risk individuals will demand less savings, the price of the bond will decrease, and low-risk individuals will save more and will be willing to pay less for the initial insurance contract. Implicit in the argument is the fact that the lower demand for savings from the riskier individuals will *not* be compensated by another agent's increase in demand. But note that when the insurance company offers better coverage for any type, it must increase its savings in order to honor the extra coverage it is promising. At least in principle, it could occur that if the firm were to demand the riskless bond in the local market, its extra demand for the bond could offset the decrease in demand by the riskier type, and the overall demand and the resulting price could be higher, in which case the low-risk individuals would be willing to pay more, not less, for their initial contract.

To determine whether this could be possible, we re-compute the equilibrium coverages of Section 7.1, under the new conditions. Here, we maintain the assumption that the level of savings demanded by the insurance firm is still given by Eq. (3). For the same parameters as before, Table 1, we obtain the equilibrium coverages of Fig. 2. The qualitative conclusion is the same as before: *even when the insurance firm demands its savings in the internal financial market, there exist economies where the two types are offered the same insurance contract at equilibrium.*

8. The mechanism

In this section we pin down the mechanism that leads to our results. We show that it is the endogeneity of the individual rationality constraint that can preclude separation and makes it

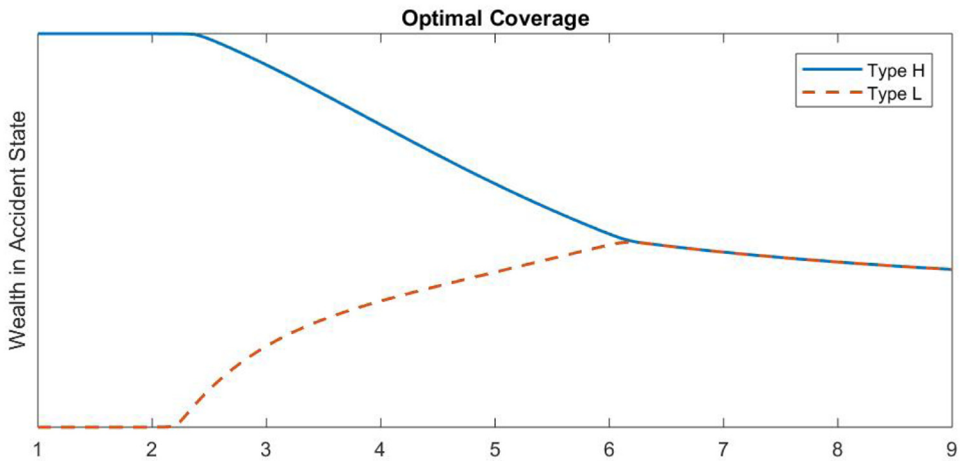

Fig. 2. Equilibrium coverages with internal funding.

suboptimal for the principal to provide efficient insurance at the top.[14]

We start by showing that separation is not precluded by incentive compatibility. In effect, our model is just like classic contract theory in the sense of incentive compatibility: it is always possible for the insurer to separate different risks with a marginal change in coverage, making profits conditional on participation. We proceed to show that the usual arguments fail exactly because participation decisions may change when the marginal change in coverage is introduced.

8.1. Single crossing of indirect utilities

We will say that *incentive separation* is possible if, for any individually-rational pooling menu, there is an incentive-compatible separating menu that dominates it and is such that all individuals prefer their contracts under this new menu. In the language of our model: incentive separation is possible if, for any pooling menu $M = \{(p, w)\}$ such that $\bar{\omega} - \lambda < w < \bar{\omega}$, there exists an incentive-compatible menu $\{(p^H, w^H), (p^L, w^L)\}$ such that, for both i, $V^i(p^i, w^i, q(w^L, w^H)) \geq V^i(p, w, q(w^L, w^H))$.[15]

The following concept extends the logic of the Spence–Mirrlees condition.

Definition 3. We say that the economy satisfies the *ordering property* if for any point (w, w), with $w \in (\bar{\omega} - \lambda, \bar{\omega})$,

$$\frac{\partial V^H}{\partial w}(p, w, q) + \frac{\partial V^H}{\partial q}(p, w, q)\frac{\partial q}{\partial w^H}(w, w) > \frac{\partial V^L}{\partial w}(p, w, q) + \frac{\partial V^L}{\partial q}(p, w, q)\frac{\partial q}{\partial w^H}(w, w)$$

where $q = q(w, w)$.

That any economy obeying this property satisfies incentive separability goes as follows. Assume that the economy is at a given pooling menu that presents coverage w. Suppose that the firm proposes a new menu which is separating, gives more coverage to the riskier individuals and keeps the low-risk type accepting w. The price of savings is changed, but if at this new price marginal utilities are still ordered, then these coverages can be incentive compatible for appropriate choices of premia. The ordering property is simply the expression for ordering marginal utilities at new prices when the new contract is arbitrarily close to w.

Our next result is that our model with endogenous reservation utilities still displays the ordering property, and hence satisfies incentive separability.

Proposition 4. *Even in the presence of financial markets, the economy satisfies the ordering property.*

Proof. This is a matter of computing derivatives. Consider a contract (p, w). By using the envelope theorem we get, for any q, that

$$\frac{\partial V^i}{\partial w}(p, w, q) = \pi^i u'(w + y^i(w, q)),$$

while, by Roy's identity,

$$\frac{\partial V^i}{\partial q}(p, w, q) = -y^i(w, q).$$

Since $\partial q/\partial w^H < 0$, by Lemma 1, it suffices to show that $\pi^H u'(w + y^H(w, q)) > \pi^L u'(w + y^L(w, q))$ for any $w \in [\bar{\omega} - \lambda, \bar{\omega}]$, given a price q, and that $y^H(w, q) > y^L(w, q)$.

Let W be the random variable paying w in the accident state and $\bar{\omega}$ otherwise. The first-order conditions of the two types' (ISP) programs imply that:

$$\mathrm{E}_H[u'(W + y^H(x, q))] = \mathrm{E}_L[u'(W + y^L(x, q))] \tag{13}$$

Let us assume, by way of contradiction, that $\pi^H u'(w + y^H(w, q)) \leq \pi^L u'(w + y^L(w, q))$. Then, as $\pi^H > \pi^L$ and because u is concave, it must be true that $y^H(w, q) > y^L(w, q)$. For (13) to be possible given the contradiction assumption, we would need

$$(1 - \pi^H)u'(\bar{\omega} + y^H(x, q)) > (1 - \pi^L)u'(\bar{\omega} + y^L(x, q)).$$

But again, because $\pi^H > \pi^L$ and by concavity, if $y^H(w, q) > y^L(w, q)$, such inequality cannot be satisfied.

14 Reservation utilities are type-dependent in any insurance problem. In our model, they are *also* endogenous. Type-dependence and endogeneity are important but distinct phenomena. Although a large literature has been devoted to the former – see Maggi and Rodriguez-Clare (1995), and Jullien (2000) –, not much has been said about the latter.

15 Notice that incentive separation does not rule out the possibility of there being pooling equilibria, as separation would also require that the deviation menu $\{(p^H, w^H), (p^L, w^L)\}$ be individually rational. In classical settings, where the reservations utilities are exogenous, incentive separation suffices for separation, naturally. Thus, in those settings any condition that guarantees incentive separation suffices for separation.

On the other hand, just notice that $u'(\bar{\omega}+y) < u'(w+y)$, for $w \in [\bar{\omega}-\lambda, \bar{\omega})$ and any $y \in \mathbb{R}$. Then, because $\pi^H > \pi^L$, $\mathrm{E}_H[u'(W+y)] > \mathrm{E}_L[u'(W+y)]$. Therefore, for the first-order conditions to hold at equilibrium, $y^H(w,q) > y^L(w,q)$ is a necessary condition. □

The proof of this proposition relies on showing that marginal utilities can be ordered at any prices. That is a consequence of the fact that, even though savings approximate the marginal utilities in the accident state between types, they are not enough to make the high-risk agents more satiated than the low-risk individuals. Otherwise, the low-risk agents would be willing to pay more than the high-risk types for one unit of the riskless bond, contradicting equilibrium in the savings market.

8.2. Endogenous outside options

Although the logic that incentive separation implies separation seems straightforward, it is not true when outside options are endogenous. In that case, building a new menu will *per se* alter reservation utilities, and it is not immediate that individual rationality constraints continue to hold. We now identify an externality that may prevent separation from occurring at equilibrium. This externality arises from the firm's understanding that the choices of menu affects the equilibrium price of savings, which impacts the agents' willingness to pay for insurance.

Recall that menu M is feasible if $E(M) \neq \varnothing$, and say that a feasible M is a "best pooling menu" if it is a solution for the firm's problem with the added constraint that it be pooling. Also, given a pooling menu $M = \{(p,w)\}$, denote by $x^i = w + y^i(w, q(w,w))$ the amount consumed by an i-type in state A; and by X^i the random variable that takes the value x^i in state A and $\bar{\omega} + y^i(w, q(w,w))$ in state N.

Again, for the sake of simplicity we first derive the result in the case where the insurance company is funded externally.

Lemma 3. *Assume that M is a best pooling menu. Let its coverage be $\tilde{w}$ and let $q = q(\tilde{w}, \tilde{w})$. There is* no *local feasible deviation $\tilde{M}$ from M such that $G(\tilde{M}, \kappa) > G(M, \kappa)$ if, and only if,*

$$\mu^H \pi^H \left[u'(w + y^H(w,q)) - \kappa\right] + \left[y^L(\bar{\omega}-\lambda, q) - y^L(w,q)\right] \frac{\partial q}{\partial w^H}(w,w) \leq 0. \tag{14}$$

In more fundamental terms, the conclusion holds true if, and only if,

$$u'(x^H) - k + \frac{u''(x^H)\left[k(\pi^L\mu^L + \pi^H\mu^H) - \pi^L u'(x^L)\right]}{\mu^H\pi^H u''(x^H) + \mu^L\pi^L u''(x^L)\frac{\mathrm{E}_H[u''(X^H)]}{\mathrm{E}_L[u''(X^L)]}} \leq 0. \tag{15}$$

This lemma follows from Lemmas 5, 6, and 7, which are presented and proven in the Appendix. It provides (locally) necessary and sufficient condition for there to be a profitable and feasible deviation from the optimal pooling menu, which is useful for understanding the following heuristics. First, note that the first term in Eq. (14) measures the distance from efficient insurance for the high-risk individual at the best pooling contract. This term is always positive, so price impacts disappear (i.e. if the insurer does not recognize the effect of the contract on the price of savings) such inequality cannot be satisfied and pooling cannot be optimal. This observation recovers the qualitative result of the benchmark framework with no financial markets.

In our setting, where the insurance company understands its effective market power, the second term in Eq. (14) is the key. By Lemma 1, this term is always negative, so the inequality in Eq. (15) may be satisfied for some economies. We submit that this term measures the general equilibrium effect on prices imposes an additional layer of costs for separation. In order to see the source of the term, assume that the economy is at a pooling menu M, with coverage $w > \bar{\omega} - \lambda$, such that the low-risk agents' individual rationality constraints are binding:[16]

$$V^L(p,w,q) - V^L(0, \bar{\omega}-\lambda, q) = 0.$$

Suppose that the monopolist considers a new menu providing more coverage to the high-risk agents and keeping the low-risks at the original contract. If this new menu is sufficiently close, the individual rationality constraint for individuals of type L is changed to

$$\left[\frac{\partial V^L(p,w,q)}{\partial q} - \frac{\partial V^L(0,\bar{\omega}-\lambda,q)}{\partial q}\right]\frac{\partial q}{\partial w^H} = \left[y^L(\bar{\omega}-\lambda,q) - y^L(w,q)\right]\frac{\partial q}{\partial w^H} < 0,$$

where we used the envelope theorem to compute the differentials of the indirect utility functions, and where the inequality comes from Lemma 1 and the fact that $w > \bar{\omega} - \lambda$. The rationale above shows that price variations lead to a violation of an individual rationality constraint.

In words, when the firm increases insurance for riskier individuals, they save less on the risk-free asset, which results in a reduction in the price of saving. Such change in prices affects the decision of the low-risk individuals between accepting their contract or not, and price variations benefit disproportionately the outcome obtained through the outside option vis-à-vis the utility of accepting the contract.[17] As a consequence, the price variation alters the attractiveness of the menu, making it less appealing for the safer agents. Of course, their insurance premium can be used by the monopolist to transfer utility to the L types so as to satisfy their participation constraint. But this is costly for the monopolist and may offset the gains extracted from the H types, in particular if their demand for assets is very inelastic and they represent a large fraction of the population.

Naturally, if the firm is trying to separate agents, it must take this additional cost into account. This is the *market effect* in our model. It tends to prevent separation by affecting the willingness of individuals to continue to accept a contract after the price of savings changes. Together, Proposition 4 and Lemma 3 identify the source of pooling as arising from individual rationality constraints, rather than from incentive compatibility.

Importantly, Proposition 2 is also a consequence of the endogeneity of the outside option. To see this, consider again Eq. (15). That equation shows that the inefficiency wedge $\kappa - u'(x^H)$ is positive *only* because prices adjust to a change in coverage "at the top". Specifically, that equation can be interpreted as follows: when the principal extends coverage (marginally) to the high-risk type, he loses κ and gains the marginal utility of the high type, $u'(x^H)$, as usual. In addition, however, when he does so he also decreases prices, generating pressure in the outside option of the low-risk type. Because of this effective extra cost to increasing coverage, coverage has got to be lower than efficient for both types.

The extension of this insight to the case of internal funding by the insurer is subtle and depends on three features of the example. First, the individuals' CARA preferences in the example display risk-aversion and prudence. Secondly, by the law of large numbers, the firm faces no uncertainty. And, finally, we are

[16] It is proved in the Appendix that this is indeed the case in the best pooling menu. See Lemma 6.

[17] This is so because the direct effect of a price change is to make savings cheaper. For any individual, her asset holding are higher when she is not covered – that is, in the outside option – than when she is. Consequently, she gains more through the devaluation of savings when she does not accept any contract.

assuming that the firm only saves the amount that is needed to remain solvent in the second period.

Under these three features, the mechanism behind our results continues to operate. Suppose that the firm contemplates and improvement of the coverage in the high-risk contract by $\mathrm{d}w^H$. Because the firm faces no risk, it will increase its bond holdings by $\pi^H \mathrm{d}w^H$, per insuree, and, by Lemma 1, the price of the asset will drop.

This guarantees that when the firm improves the coverage offered to the high-risk agents, their demand for savings decreases by more than the extra savings that the firm itself will need to constitute. Since the firm does not over-save in order to manipulate the price, the aggregate demand for savings decreases, the price of the bond falls, and the high-risk individuals reduce their willingness to pay for insurance. If this effect is significant enough, the price concession that the firm would need to give to retain the insurance demand by the low-risk insurees may prevent it from improving the contract offered to the high-risk types, hence choosing to remain in a pooling (and inefficient) situation.

9. Monopolistic banker

While the assumption of a monopolistic insurer is not uncommon in the contract theory literature, it is less innocuous in our setting, where the choice of an insurance menu is allowed to have impact on the price of the financial instruments. To counteract this effect, suppose now that, unlike so far, the financial market consists of a monopolistic supplier of assets. Whether our previous results are robust to this modification depends on how asset prices react to an increase in insurance coverage. If they react negatively, just as in our original model, our original rationale continues to hold. We first assume that ${}^{dq}/_{dw^i} < 0$ and argue why our results are qualitatively robust to this extension. Then, we provide sufficient conditions for the assumption to be satisfied.

For shortening notation, let $y^i(q)$ be the demand for savings for type $i \in \{L, H\}$ as a function of savings price q. Function y^i is, of course, a function of coverage, but from the point of view of the banker it is just a function of prices — i.e., the banker does not take into account the effect of his strategy on the insurance market. In period the present, the monopolist collects all the expenses of the insurees in savings: $q \sum_{i\in\{L,H\}} \mu^i y^i(q)$. In the future, he must honor the return of the asset. Now, the monopolist banker understands how y^i changes with q, so his profit maximization problem is

$$\max_q \left\{ (q-1) \sum_{i\in\{L,H\}} \mu^i y^i(q) \right\}. \tag{16}$$

Necessary conditions for the resulting asset price are that

$$\sum_{i\in\{L,H\}} \mu^i y^i(q) + (q-1) \sum_{i\in\{L,H\}} \mu^i \frac{dy^i(q)}{dq} = 0, \tag{17}$$

which is the classic monopolistic result of unit elasticity of demand (considering price $q-1$ and no cost).

For this fixed coverage, let q^* be the competitive price, so $\sum_i \mu^i y^i(q^*) = 0$. The following result proves existence of a solution to the banker's problem and gives sufficient conditions for the dependence of equilibrium asset prices on insurance coverage.

Lemma 4. *A solution q exists to the banker's problem, and it satisfies* $q \in [\min\{1, q^*\}, \max\{1, q^*\}]$. *Moreover, if index u is prudent and* $u'(\bar{w}) > 1$, *then* ${}^{dq}/_{dw^i} < 0$.

Concluding remarks

In terms of its topic, this paper studies the interaction between insurance and financial markets; in terms of modeling, it lies in the intersection of contract theory and general equilibrium theory. Individuals who differ only in risk have access to insurance contracts offered by a monopolist and can also save through a competitive market — the former take the standard form from contract theory, the latter is modeled as a Walrasian market. Two substantial results arise from this interaction: the equilibrium menu of insurance contracts always displays under-insurance of *all* agents, even the riskiest ones, and it may even pool agents of different riskiness together.

To be sure, ours is not the first paper where the agent's ability to take actions in addition to her contract upsets the classical results of contract theory. The mechanism through which this occurs here, however, differs from the well-understood insight of the hidden action literature. In the canonical setting with no hidden actions, Spence–Mirrlees conditions imposed on the preferences of the agents drive the main results, through their implications on the incentives of the agents to misrepresent their types. If in addition to contracting with the principal, the agents can also play hidden actions, the Spence–Mirrlees conditions imposed on the primitives of the problem need not suffice for the same type of single-crossing condition to hold on the indirect utilities of the agents, which is the relevant object, from the point of view of the principal, after the agents take their optimal hidden actions. If the resulting indirect utilities fail to display single crossing, the incentive compatibility conditions do not suffice to preclude, say, pooling at equilibrium. But whether on the primitives or on the indirect utilities, the chief mechanism determining the type of equilibrium is one of self-selection and operates through the incentive compatibility conditions.

In this paper, the mechanism is different. In addition to their own hidden actions, all the agents are affected by an aggregate statistic arising from the whole profile of actions in the economy: the price of savings. This introduces a new externality: by changing the contract meant for one type of agent, the principal affects the action of that agent in a direct manner and, indirectly, through the aggregate statistic, the actions of all agents. This externality is internalized by the principal and affects his optimal decision. In particular, a change that would make the contract of a particular type more profitable if the actions remained fixed may cause, through the indirect effect, that another type of agent find it beneficial to deviate from the contract that was initially meant for her. Simply put, a deviation that raises more profits from a type may reduce the profits earned from the interaction with another type. For the case considered in this paper, where the aggregate is the equilibrium price of a competitive equilibrium, we show that the indirect effect upsets the classical properties of optimal contracts in other settings: the principal may find it optimal to refrain from offering an efficient contract to even the type that values insurance the most (and may even choose to pool types).

Some robustness exercises suggest that the mechanism is more general than our formal results: qualitatively, they hold also in the presence of a non-competitive supplier in the savings market, and when there is competition in the insurance market.

Acknowledgments

Work on this paper started when both authors were at EPGE-FGV. We benefited from comments given by audiences at multiple conferences and seminars, and in particular by Braz Camargo, Humberto Moreira and Martine Quinzii.

Appendix. Lemmata

Proof of Lemma 1. By Inada, both Program (ISP) have interior solution. The first-order conditions of the former problem are that

$$q = \pi^i u'(w^i + y^i) + (1 - \pi^i)u'(\bar{\omega} + y^i). \qquad (*)$$

By strict concavity and the implicit function theorem, function y^i is continuously differentiable and strictly decreasing.

The first-order conditions that characterize function $q(w)$, from the maximization of (4) subject to (5) are that Eq. $(*)$ hold for both types and $\mu^L y^L + \mu^H y^H + Y(w) = 0$. Differentiating this system with respect to w^L gives

$$\begin{pmatrix} -1 & D^2U^L & 0 \\ -1 & 0 & D^2U^H \\ 0 & \mu^L & \mu^H \end{pmatrix} \begin{pmatrix} \mathrm{d}q \\ \mathrm{d}y^L \\ \mathrm{d}y^H \end{pmatrix} = \begin{pmatrix} -\pi^L u''(w^L + y^L) \\ 0 \\ -\mu^L \pi^L \end{pmatrix} \mathrm{d}w^L,$$

where

$$D^2U^i = \pi^i u''(w^i + y^i) + (1 - \pi^i)u''(\bar{\omega} + y^i).$$

By the implicit function theorem, q is differentiable with respect to w^L and

$$\frac{\partial q}{\partial w^L}(w) = \frac{\mu^L \pi^L D^2 u^H [u''(w^L + y^L) - D^2 u^L]}{\mu^L D^2 u^H + \mu^H D^2 u^L}.$$

By strict concavity, D^2u^H and D^2u^L are both negative. By prudence, moreover,

$$u''(w^L + y^L) - D^2u^L = (1 - \pi^L)[u''(w^L + y^L) - u''(\bar{\omega} + y^L)] < 0.$$

It follows that $\partial q/\partial w^L(w) < 0$, and the same argument applies to the derivative with respect to w^H. $\square$

Proof of Lemma 2. Individual rationality for the riskier type, H, is redundant. Next, assume that the incentive compatibility constraint for individuals of type L is not binding. Then, the argument of the following proposition closely parallels the characterization of an optimal contract in the canonical model of contract theory, and its details thus be omitted: Eq. (7) guarantees that efficient insurance is achievable;[18] then, the three claims in the proposition follow simply by taking first-order conditions of the monopolist's problem.

Of course, to complete the argument we need to show that, indeed, $U^L(p^L, w^L) \geq U^L(p^H, w^H)$ at the solution of the relaxed problem above. Explicitly, this requires that $P_H - P_L \geq \pi_L[u(w_H) - u(w_L)]$. By the second part of the proposition, $P_H - P_L = \pi_H[u(w_H) - u(w_L)]$, which implies, since $\pi_H > \pi_L$, that we just need to show that $w_H \geq w_L$. The third part of the proposition implies that this is the case, since u is concave. $\square$

Lemma 5. *For any q and $\bar{\omega} - \lambda \leq w \leq \bar{\omega}$, let $\Delta V^i(p, w) = V^i(p, w, q) - V^i(0, \bar{\omega} - \lambda, q)$. Then, $\Delta V^H(p, w) > \Delta V^L(p, w)$.*

Proof. By the envelope theorem, we can write $\Delta V^i(p, w) = -p + \int_{\bar{\omega}-\lambda}^{w} \pi^i u'(x + y^i(x, q))\mathrm{d}x$. But then, by Claim 1, we have that, $\pi^H u'(x + y^H(x, q)) > \pi^L u'(x + y^L(x, q))$. Using this information,

$$\Delta V^H(p, w) = -p + \int_{\bar{\omega}-\lambda}^{w} \pi^H u'(x + y^H(x, q))\mathrm{d}x > -p + \int_{\bar{\omega}-\lambda}^{w} \pi^L u'(x + y^L(x, q))\mathrm{d}x = \Delta V^L(p, w). \quad \square$$

18 In the absence of the assumption on marginal utilities, this proposition only changes in that market exclusion can be optimal. That is, when $u'(\bar{\omega} - \lambda) < \kappa$, it makes no sense to provide insurance for any of the agents, then both are excluded from the market. Besides, if $u'(\bar{\omega}) > \kappa$, we would have $w^H = \bar{\omega}$.

Lemma 6. *If a best pooling menu $M = \{(p, w)\}$ consists of an interior contract, it satisfies*

$$\pi^L u'(w + y^L(w, q)) + \left[y^L(\bar{\omega} - \lambda, q) - y^L(w, q)\right] Dq = \kappa(\pi^L \mu^L + \pi^H \mu^H).$$

where $q = q(w, w)$ and $Dq = \partial q/\partial w^L + \partial q/\partial w^H$. Moreover, $p = \Delta V^L(0, w)$.[19]

Proof. Solving for the best pooling menu consists of facing the problem of the firm when the latter is searching for optima in the space of pooling contracts. In that space, incentive constraints hold trivially. Additionally, by Lemma 5, in such space the individual rationality constraint for the riskiest type is redundant. Therefore, (FMP) can be rewritten as

$$\max_{p,w} \left\{ p - v \sum_i \mu^i \pi^i w : -p + V^L(0, w, q(w, w)) \geq V^L(0, \bar{\omega} - \lambda, q(w, w)) \right\}.$$

Because we know that, for fixed coverage, q is a constant, it is always in the interest of the firm to set equality in the individual rationality constraint of the L-types, which establishes the second claim in the lemma.

The first claim then follows from taking the necessary first order conditions of the problem above, assuming interiority.[20] $\square$

Lemma 7. *Let $\mathbf{A}$ be a matrix with dimensions $K \times L$, with $(\mathbf{A}_k)_{k \leq K}$ its rows, and $\mathbf{b} \in \mathbb{R}^L$. Suppose that, for the matrix $\bar{\mathbf{A}} = (\mathbf{A}_k)_{2 \leq k \leq K}$, there is $z \in \mathbb{R}^L$ such that $\bar{\mathbf{A}} z \geq \bar{\mathbf{b}}$, where $\bar{\mathbf{b}} = (\bar{\mathbf{b}}_k)_{2 \leq k \leq K}$. Then, one and only one of the following two statements is true:*

(A) There exists a vector $z \in \mathbb{R}^L$ such that $\mathbf{A}z \geq \mathbf{b}$ with $\mathbf{A}_1 z > \mathbf{b}_1$; or

(B) There exists a vector $\gamma \in \mathbb{R}^K_+$ such that $\gamma_1 > 0$, and with the property that $\sum_k \gamma_k \mathbf{A}_k = 0$ and $\sum_k \gamma_k \mathbf{b}_k \geq 0$.

Proof. This is just a minor variation of Farkas's Lemma. See Theorem 22.2 in Rockafellar (1970). $\square$

Proof of Lemma 3. This lemma follows from Lemmas 5, 6, and 7, but we first add some notation to avoid cumbersome expressions. Define $u_i' = u'(x^i)$, $\Delta y^i = y^i(\bar{\omega} - \lambda, q) - y^i(w, q)$, $\Delta V^i = V^i(p, w, q) - V^i(0, \bar{\omega} - \lambda, q)$ and $\partial_i q = \partial q/\partial w^i(w, w)$.

As in Proposition 4, if a deviation is close enough to the original contract, it must satisfy in differential form all the constraints. If it is profitable for the firm, then it must also satisfy

$$\mu^L(\mathrm{d}p^L - \pi^L \kappa \mathrm{d}w^L) + \mu^H(\mathrm{d}p^H - \pi^H \kappa \mathrm{d}w^H) > 0.$$

Now, notice that incentive compatibility constraints are binding at the original menu, M. Therefore, for them to hold in the deviation menu, for each type,

$$-\mathrm{d}p^i + \frac{\partial V^i(0, w, q)}{\partial w}\mathrm{d}w^i \geq -\mathrm{d}p^j + \frac{\partial V^i(0, w, q)}{\partial w}\mathrm{d}w^j,$$

given that the derivatives of V^i relative to prices appear on both sides of the inequality and, hence, cancel out. By the envelope theorem, the previous inequality can be written as

$$-\mathrm{d}p^i + \pi^i u_i'(x^i)\mathrm{d}w^i \geq -\mathrm{d}p^j + \pi^i u_i'(x^i)\mathrm{d}w^j.$$

19 That is, $\Delta V^L(p, w) = 0$.

20 Without interiority, the same condition holds with the left hand side being less or equal to the right hand side.

For individual rationality, we must have, for each i,[21]

$$-\mathrm{d}p^i + \pi^i u'(x^i)\mathrm{d}w^i + \Delta y^i(\partial_i q \mathrm{d}w^i + \partial_j q \mathrm{d}w^j) \geq -\Delta V^i.$$

It is possible to arrange this system of five linear inequalities in matrix form, as $\mathbf{A}z \geq \mathbf{b}$ where

$$\mathbf{A} = \begin{pmatrix} \mu^H & \mu^L & -\mu^H\pi^H\kappa & -\mu^L\pi^L\kappa \\ -1 & 1 & \pi^H u'_H & -\pi^H u'_H \\ 1 & -1 & -\pi^L u'_L & \pi^L u'_L \\ -1 & 0 & \partial_H q \Delta y^H + \pi^H u'_H & \partial_L q \Delta y^H \\ 0 & -1 & \partial_H q \Delta y^L & \partial_L q \Delta y^L + \pi^L u'_L \end{pmatrix},$$

$z = (\mathrm{d}p^H, \mathrm{d}p^L, \mathrm{d}w^H, \mathrm{d}w^L)'$ and $\mathbf{b} = (0, 0, -\Delta V^H, 0)'$.

We want to find a necessary and sufficient condition for the existence of γ in statement (B) of Lemma 7. First of all, notice that the hypothesis on the statement of Lemma 7 is satisfied: $\bar{\mathbf{A}}0 \geq \bar{\mathbf{b}}$. Now, assume there is a γ satisfying (B), that is, the pooling contract allows no local deviation. We shall show that this is so whenever (p, w) satisfies Eq. (15), and vice-versa.

By the second part of property (B) we know that $\gamma\mathbf{b} = \gamma_4(-\Delta V^H) \geq 0$. By Lemma 5, $\Delta V^H > \Delta V^L = 0$, where the last equality is a consequence of Lemma 6, since we are assuming (p, w) to be a best pooling contract that is interior. Therefore, because $\gamma_4 \geq 0$, it must be the case that $\gamma_4 = 0$.

Write as $\mathbf{A}^\ell$, for $\ell = 1, \ldots, 4$, the columns of matrix $\mathbf{A}$. By statement (B) in Lemma 7, $\gamma\mathbf{A}^\ell = 0$ for all ℓ. By taking $\gamma(\mathbf{A}^1 + \mathbf{A}^2) = 0$ we get $\gamma_1 = \gamma_5$. Substituting that in $\gamma\mathbf{A}^1$, we also get $\gamma_2 = \gamma_1\mu^H + \gamma_3$. Taking $\gamma(\mathbf{A}^3 + \mathbf{A}^4) = 0$ and using $\gamma_1 = \gamma_5$ we get that

$$\gamma_1 \cdot \left[\pi^L u'_L + (\partial_L q + \partial_H q)\Delta y^L - \kappa(\pi^L\mu^L + \pi^H\mu^H)\right] = 0.$$

Because we are assuming (p, w) to be a best interior pooling contract, the term in braces is zero by Lemma 6. Therefore, this equation is satisfied automatically. So, the last equation to be checked is either $\gamma\mathbf{A}^3 = 0$ or $\gamma\mathbf{A}^4 = 0$. Choosing the first one gives us

$$\gamma_1 \left[\mu^H\pi^H(u'_H - \kappa) + \partial_H q \Delta y^L\right] = \gamma_3(\pi_l u'_L - \pi^H u'_H).$$

As we proved in Lemma 1, $\pi^L u'_L - \pi^H u'_H < 0$, so that for γ_1 and γ_3 to be greater or equal to zero, it must be true that

$$\mu^H\pi^H(u'_H - \kappa) + \partial_H q \Delta y^L \leq 0. \tag{18}$$

In order to conclude the proof of sufficiency, it suffices to show that the inequality in Eq. (18) is equivalent to the one in Eq. (15). That is done by using the necessary condition a best pooling contract must satisfy by Lemma 6 to substitute for Δy^L in the above inequality.

Note that by rearranging the expression in Lemma 3,

$$\Delta y_L = \frac{k(\pi^L\mu^L + \pi^H\mu^H) - \pi^L u'(x_L)}{\partial_H q + \partial_L q}.$$

Plugging this into Eq. (18), we have that

$$\mu^H\pi^H(u'(x_H) - k) + \left[k(\pi^L\mu^L + \pi^H\mu^H) - \pi^L u'(x_L)\right]\frac{\partial_H q}{\partial_H q + \partial_L q} \leq 0 \tag{19}$$

Now, we just need to calculate $\partial_i q$. Recall first order condition for type i is that $\mathrm{E}_i\, u'(X_i) = q$. After a change in w_i, this equation changes so that

$$dq = \pi^i u''(x^i)dw^i + \mathrm{E}_i[u''(X^i)]dy^i$$

Similarly, with the first order condition for type j, we get that $dq = \mathrm{E}_j[u''(X^j)]dy^j$ and, by market clearing, $\mu^H\mathrm{d}y^H + \mu^L\mathrm{d}y^L = 0$. We can solve these last three equations, we get that

$$\partial_i q = \pi^i u''(x^i)\frac{\mu^i\,\mathrm{E}_j[u''(X^j)]}{\mu^i\,\mathrm{E}_j[u''(X^j)] + \mu^j\,\mathrm{E}_i[u''(X^i)]}.$$

Then,

$$\frac{\partial_H q}{\partial_H q + \partial_L q} = \mu^H\pi^H\frac{u''(x^H)}{\mu^H\pi^H u''(x^H) + \mu^L\pi^L u''(x^L)\frac{\mathrm{E}_H[u''(X^H)]}{\mathrm{E}_L[u''(X^L)]}}.$$

Plugging this result into Eq. (19), and dividing by $\pi^H\mu^H > 0$, we get Eq. (15).

Notice, however, that necessity is also proved. If (p, w) is a best pooling contract that allows no local deviations that are profitable for the firm, then a γ satisfying the conditions in (B) must exist. Therefore, Eq. (18) must hold. □

[21] We could rightfully ignore individual rationality for the H-types. We opt not to do so just to maintain everything clear.

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