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Journal of Economic Theory 146 (2011) 1569–1597

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Idiosyncratic risk and financial policy ☆

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Received 1 May 2007; final version received 8 October 2010; accepted 4 January 2011

Available online 3 March 2011

Abstract

In economies subject to uninsurable idiosyncratic risks, competitive equilibrium allocations are constrained inefficient: reallocations of assets support Pareto superior allocations. This is the case even if the asset market for the allocation of aggregate risks is complete.
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JEL classification: D52; D60; H20

Keywords: Uninsurable idiosyncratic risks; Constrained suboptimality

Idiosyncratic risk does not affect the allocation of resources at Pareto optimal allocations.[1] Competitive equilibrium allocations inherit this property if the asset market for the insurance of idiosyncratic risk is complete. But, if realizations of idiosyncratic shocks are publicly unob-

☆ We wish to thank a referee and an associate editor whose comments helped us understand our paper better and, we hope, improve it. We also want to thank Pablo Beker, David Cass, Julio Dávila, John Geanakoplos, Peter Hammond, Felix Kübler, Michael Magill, Michael Mandler, Martine Quinzii, Paolo Siconolfi, Tony Smith, and participants in the First CARESS-Cowles Conference on General Equilibrium Theory, the XIV European Workshop on General Equilibrium Theory, the 2005 NSF/CEME Mathematical Economics Conference in Honor of Gerard Debreu, the IMPA Conference in Honor of Aloisio Araujo, the CRETA Workshop in Honor of Peter Hammond and the Mathematical Economics Workshop at Yale, for interesting and helpful comments. This work was carried while Carvajal was at the Cowles Foundation for Research in Economics, Yale University, whose generous hospitality and financial support he gratefully acknowledges.

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[1] See [3,16,17].

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doi:10.1016/j.jet.2011.03.012

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servable or unverifiable, idiosyncratic risk may well not be insurable; indeed, this is a standard assumption in macroeconomics.[2]

In abstract, two-period economies where financial markets are incomplete, competitive equilibrium allocations are constrained inefficient, *generically in the spaces of economies*, as demonstrated by [10].[3] This result, however, does not imply that constrained inefficiency holds for economies with idiosyncratic risk, as the particular structure of these economies is non-generic: the economies considered in the literature on incomplete financial markets allow for individual perturbations of preferences and endowments; but if one is to distinguish between idiosyncratic and aggregate risk, one must impose that some individuals, given an aggregate shock, face exactly the same idiosyncratic shocks and have the same preferences.

Thus, there is a gap between the type of model usually considered in macroeconomics and the generic results obtained in the literature on incomplete financial markets. The goal of this paper is to illustrate how these two strands of literature can be brought together. We do this in two ways. On one hand, we show that in the set of two-period economies with uninsurable idiosyncratic risk, competitive equilibrium allocations are, indeed, generically constrained suboptimal: reallocations of assets support Pareto superior allocations. In other words, we extend the generic results from the incomplete markets literature to the type of risk that is of most interest in macroeconomics: in most economies where idiosyncratic risk is uninsurable, a reallocation of the financial assets that permit insurance against aggregate risk can be used to make all types of individuals in the economy *ex-ante* better off. Importantly, this result does not depend on the assumption that there exists some aggregate shock against which the individuals cannot insure.[4]

On the other hand, we use a series of examples to emphasize the mechanism by which a reallocation of assets brings about a Pareto improvement: the ability of the policy to perturb future relative prices. This mechanism, which lies at the core of our general argument, is made explicit in an example for an exchange economy with ex-ante heterogeneous consumers, but also in two-period production economies with ex-ante homogeneous individuals. In this latter case, we show that the presence of uninsurable idiosyncratic risk leads to inefficiently high levels of savings at equilibrium, under standard assumptions. Importantly, the same mechanism we show operates in an economy of overlapping generations with individuals who are homogenous when young but face uninsurable, idiosyncratic labor shocks when old. Of course, in this type of economy equilibrium allocations are subject to the standard problem of dynamic inefficiency, even under certainty.[5] What our example shows is how, in the presence of idiosyncratic risk, the ability to perturb relative prices through perturbations to savings decisions must be compounded with standard recommendation derived from the problem of dynamic inefficiency. Importantly, in an economy in which the equilibrium interest rate is below the rate of population growth, a policy that forces individuals to invest more (the standard Golden Rule recommendation) may leave all generations worse off in lifetime utility.

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[2] See [14] and [13].

[3] Also, [6] and [7].

[4] In fact, for the sake of simplicity, we prove our results for the case in which there is *only* idiosyncratic risk. But it is important to note that the result also holds in the presence of aggregate risk, and that the mathematical argument would be, as a matter of fact, simpler in that case, although it would require a more complicated notation. Also, the result holds whether the asset market for the allocation of aggregate risk is complete or not.

[5] See [9].

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This paper is related to the work of [8], where an infinite-horizon, one-sector neoclassical economy is considered, under the assumption that idiosyncratic income and wealth shocks are uninsurable. There, in a calibration to US data, it is shown that at competitive equilibrium the economy underinvests in steady state.[6] It is also related to [11], where a two-period economy with idiosyncratic risk is used to study the welfare effects of taxation of labor and capital income, which occur through its impact on capital accumulation.

The demonstration that competitive equilibria in economies with uninsurable idiosyncratic shocks are constrained suboptimal makes an important methodological point relevant for economic policy. Intervention is often said to be counterproductive because competitive equilibrium cannot be Pareto improved; our theorem shows that such a view is untenable.[7]

The paper is organized as follows: The first section presents two examples that illustrate the mechanism by which the general result of constrained suboptimality holds. The first of these examples is for an economy exactly like the one for which we will prove general results, while the second example extends the analysis for the case of a storage economy. The following two sections introduce the general kind of economies in which our analysis holds, and define competitive equilibrium and Pareto efficiency for this kind of economies. Section 4 introduces the definition of constrained inefficiency for economies with uninsurable idiosyncratic risk, and states the main theorem, whose proof requires a construction and argument that are given in Section 5. Then, Section 6 presents two more examples of constrained suboptimality; the first extends the analysis to an economy with a production technology more general than storage, while the second example considers an economy of overlapping generations, and briefly assesses the extent to which the classical *Golden Rule* applies in the presence of idiosyncratic risk. A technical appendix completes the paper.

1. Some examples

Two simple examples that illustrate the main result of the paper: in the absence of insurance opportunities for idiosyncratic risk, competitive markets typically induce constrained suboptimal allocations of commodities. The first example considers a simplified economy with two types of individuals, one of whom faces uninsurable idiosyncratic risk; the example is simple enough to allow for the computation of a closed-form solution that illustrates the mechanism by which a Pareto improvement may be induced after an asset reallocation policy. The second example illustrates how this result can be extended to the case where a storage technology is available; this example is presented for general preferences first, but a specific functional form is used, again, for the purpose of obtaining an explicit solution. Storage allows constrained suboptimality to prevail even in the absence of (ex-ante) heterogeneity among individuals. Storage or, more generally, production are not encompassed by the abstract model that follows. But, importantly, the logic that underlies constrained suboptimality remains unchanged and the examples can be understood as leads for further work.

6 For other values of the parameterization, they show that at equilibrium the economy will overinvest. For related work, see [2], who studies the effects of asset market incompleteness on the equilibrium levels of interest rates and capital stock.

7 The identification of Pareto improving asset reallocations from market data remains an issue: because of the particular aggregation structure of idiosyncratic shocks, neither the positive identification results of [15] and [4], nor the negative results of [5] apply.

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1.1. A heterogeneous exchange economy

One commodity is exchanged and consumed in the first period, date 0, while two commodities are exchanged and consumed at date 1, the second period. Quantities of the date-zero commodity are x; for simplicity, the two commodities of date 1 are $l = a, b$ and their quantities x_a and x_b.

There are two types of individuals, $i = \alpha, \beta$, and each type consists of a continuum of individuals of unit mass. The intertemporal utility function of an individual of type β is

$$u^\beta(x, x_a, x_b) = x + (1-\gamma)\ln x_a + \gamma \ln x_b,$$

where, $0 < \gamma < 1$, and his endowment at date 1 consists of b units of commodity b.[8] The intertemporal utility function of an individual of type α is

$$u^\alpha(x, x_a, x_b) = x + \gamma \ln x_a + (1-\gamma)\ln x_b,$$

and his endowment at date 1 consists only of commodity a; but, importantly, it is subject to idiosyncratic shocks: it is $a \pm \varepsilon$, with equal probability.[9]

Importantly, at date 0 individuals can trade only in the consumption good and a risk-free bond that matures at date 1. The consumption good be numèraire in the first period, and q is the price of the bond. At date 1, individuals only trade on the two commodities; commodity a is numèraire, and the price of commodity b is p.

By direct computation, if holdings of the bond are y for individuals of type α and $-y$ for individuals of type β, the equilibrium price at date 1 is

$$p(y) = \frac{(1-\gamma)a + (1-2\gamma)y}{(1-\gamma)b}, \tag{1}$$

which depends non-trivially on asset holdings as long as $\gamma \neq 1/2$, a condition that we now impose. At date 1, then, the marginal utility of revenue for individuals of type β is

$$\lambda^\beta = \frac{1}{pb - y},$$

while for individuals of type α it varies with the realization of the idiosyncratic shock – the personal state of an individual – and is

$$\lambda^\alpha(\varepsilon) = \frac{1}{a + \varepsilon + y}$$

or

$$\lambda^\alpha(-\varepsilon) = \frac{1}{a - \varepsilon + y},$$

with equal probability. The optimization of individuals of type β at date 0 requires, therefore, that

$$q = \frac{1}{pb - y} = \frac{(1-\gamma)}{(1-\gamma)a - \gamma y},$$

[8] With quasi-linear preferences, it is not necessary to specify the endowments of individuals at date 0.

[9] At date 1, equal proportions of individuals of type α have endowments $a + \varepsilon$ and $a - \varepsilon$, and, as a consequence, the aggregate endowment of commodity a is guaranteed to be a, without any risk.

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while optimization of individuals of type α requires that

$$q = \left(\frac{1}{2}\right)\frac{1}{a+\varepsilon+y} + \left(\frac{1}{2}\right)\frac{1}{a-\varepsilon+y} = \frac{a+y}{(a+y)^2-\varepsilon^2}.$$

As a consequence, at equilibrium,

$$y = \frac{-a+\sqrt{a^2+4\varepsilon^2(1-\gamma)}}{2}. \tag{2}$$

A policy intervention perturbs assets holdings and makes revenue transfers at date 0: policy is a pair $(\mathrm{d}x, \mathrm{d}y)$ of transfers of revenue and bonds to individuals of type α. The welfare effects of a policy are

$$\mathrm{d}u^\alpha = \mathrm{d}x + q\,\mathrm{d}y - \frac{1}{2}\big(\lambda^\alpha(\varepsilon)x_b^\alpha(\varepsilon) + \lambda^\alpha(-\varepsilon)x_b^\alpha(-\varepsilon)\big)p'\,\mathrm{d}y,$$

and

$$\mathrm{d}u^\beta = -\mathrm{d}x - q\,\mathrm{d}y - \lambda^\beta\big(x_b^\beta - b\big)p'\,\mathrm{d}y.$$

Pareto improving interventions exist if the matrix

$$\begin{pmatrix} 1 & q - \frac{1}{2}(\lambda^\alpha(\varepsilon)x_b^\alpha(\varepsilon) + \lambda^\alpha(-\varepsilon)x_b^\alpha(-\varepsilon))p' \\ -1 & -q - \lambda^\beta(x_b^\beta - b)p' \end{pmatrix} \tag{3}$$

is nonsingular, which is the case for *any* $\varepsilon \neq 0$.[10]

In order to find the type of policy that induces a Pareto improvement, we write

$$\mathrm{d}y = \frac{\mathrm{d}u^\alpha + \mathrm{d}u^\beta}{(\Lambda^\alpha + \Lambda^\beta)p'},$$

where

$$\Lambda^\alpha = -\frac{1}{2}\big(\lambda^\alpha(\varepsilon)x_b^\alpha(\varepsilon) + \lambda^\alpha(-\varepsilon)x_b^\alpha(-\varepsilon)\big)$$

and

$$\Lambda^\beta = -\lambda^\beta\big(x_b^\beta - b\big).$$

By direct computation, $\Lambda^\alpha + \Lambda^\beta > 0$ if, and only if,

$$\frac{1}{p} < \frac{b}{pb-y}, \tag{4}$$

which is the case since $y > 0$. It follows, then, that the sign of a Pareto improving $\mathrm{d}y$ is the same as the sign of p', namely positive if $\gamma < 1/2$, and negative if $\gamma > 1/2$.

[10] Singularity of the matrix would occur if, and only if,

$$\frac{1}{2}\lambda^\alpha(\varepsilon)x_b^\alpha(\varepsilon) + \frac{1}{2}\lambda^\alpha(-\varepsilon)x_b^\alpha(-\varepsilon) = -\lambda^\beta\big(x_b^\beta - b\big),$$

which is equivalent to

$$\frac{1-\gamma}{p} = -\frac{1}{pb-y}\left(\frac{\gamma(pb-y)}{p} - b\right),$$

and, hence, to $y = 0$, which occurs only in the absence of idiosyncratic shocks, with $\varepsilon = 0$.

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1.2. A storage economy

The setting is again of two-periods, but, now, there is only a continuum of ex-ante identical individuals. One commodity is available for exchange and consumption in date 0, and, importantly, it is storable. At date 1 there are two commodities: individuals trade and consume what they have stored of the first commodity, along with any further endowment they may have, and they also exchange and consume a second commodity.

The amount of commodity that is stored by each individual at date 0 is k, and e_2 is the endowment of commodity 2 they all receive at date 1. But assume the endowment of commodity 1 is subject to idiosyncratic risk: for an individual in personal state s, the endowment of commodity 1 is $e_{1,s}$. The proportion of individuals in state s is $\pi_{i,s}$ and, for simplicity of notation, $e_1 = \mathrm{E}[e_{1,s}]$.[11]

With Bernoulli utility indices v_s, the individual's von Neumann–Morgenstern ex-ante utility is

$$u(k, x) = -k + \mathrm{E}\big[v_s(x_s)\big],$$

where $x_s = (x_{1,s}, x_{2,s})$ denotes the individuals' consumption at date 1 in state s. If commodity 1 is numèraire at date 1, so that prices can be denoted by $(1, p)$, then the nominal wealth of an individual in personal state s is $e_{1,s} + k + pe_2$. If the marginal utility of income in personal state s is λ_s, then the first-order condition for optimization at date 0 is that $\mathrm{E}[\lambda_s] = 1$, and, as a consequence, the ex-ante utility impact of an infinitesimal perturbation to the level of savings, $\mathrm{d}k$, around its competitive equilibrium level, is

$$\mathrm{d}u = -\mathrm{d}k + \mathrm{E}\big[\lambda_s\big((-x_{2,s} + e_2)\,\mathrm{d}p + \mathrm{d}k\big)\big] = \mathrm{E}\big[\lambda_s(-x_{2,s} + e_2)\big]\,\mathrm{d}p. \tag{5}$$

Under certainty, market-clearing implies that $\mathrm{E}[\lambda_s(-x_{2,s} + e_2)] = 0$, so it is impossible to improve ex-ante utility by the implementation of levels of savings different from the ones chosen under competitive equilibrium. But, if date 1 spot prices depend on the effective endowment of commodity 1, then the equilibrium utility level can be improved upon if $\mathrm{E}[\lambda_s(-x_{2,s} + e_2)] \neq 0$.

As in the previous example, a simple structure allows for a closed-form solution. First, the Bernoulli indices are state-independent and are defined by

$$v(x_s) = \ln x_{1,s} + \ln x_{2,s}.$$

By direct computation, given savings of k, equilibrium prices are

$$p = (e_1 + k)/e_2, \tag{6}$$

an expression that depends positively on k. Also, then, $\lambda_s = 2/(e_{1,s} + e_1 + 2k)$ and

$$e_2 - x_{2,s} = \frac{e_2}{2}\left(\frac{e_1 - e_{1,s}}{e_1 + k}\right),$$

so that, if, further there are only two personal states, $e_{1,s} = e_1 \pm \varepsilon$, and these states occur with equal probability, then

$$\mathrm{E}\big[\lambda_s(e_2 - x_{2,s})\big] = \frac{1}{e_1 + k}\left(\frac{\varepsilon^2}{4(e_1 + k)^2 - \varepsilon^2}\right).$$

[11] Throughout the paper, the probability measure with respect to which the expectation is taken is left implicit; that is $\mathrm{E}[e_{1,s}] = \mathrm{E}_\pi[e_{1,s}]$.

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Now, at the equilibrium level of savings, it must be true that $\mathrm{E}[\lambda_s] = 1$. By direct computation, this implies that

$$\mathrm{E}\big[\lambda_s(e_s - x_{2,s})\big] = \frac{\varepsilon^2}{4(e_1 + k)^2} > 0, \tag{7}$$

which means that in this economy individuals *underinvest* at competitive equilibrium.

1.3. The mechanism

Note the two features that are necessary for the existence of Pareto improving policies in the first example, by the looking at Eq. (3). First, it requires that the relative price depend nontrivially on the date-1 wealth of the individuals, namely that $p' \neq 0$. Second, it requires that

$$\left(\frac{1}{2}\left(\lambda^\alpha(\varepsilon)x_b^\alpha(\varepsilon) + \frac{1}{2}\lambda^\alpha(-\varepsilon)x_b^\alpha(-\varepsilon)\right)\right) + \lambda^\beta\big(x_b^\beta - b\big) \neq 0,$$

for otherwise the matrix is singular. In words, effectiveness of the policy requires that the aggregate, across individual states and types of individuals, of individual commodity trades weighted by the product of the marginal utility of income and the probability of the individual states does not vanish.

Since we can compute a closed-form solution for that economy, we can directly observe that these two requirements are satisfied generically in the space of economies: (i) from Eq. (1), one has that the first condition hods generically on preferences (as long as $\gamma \neq 1/2$); and (ii) it follows from Eq. (2) and Eq. (4) that the second condition holds generically on endowments (as long as $\varepsilon \neq 0$).

This example corresponds precisely to the general result that is presented below. The second example illustrates how this result can be extended to an ex-ante homogeneous economy with storage, through the possibility of a Pareto improving perturbation to the equilibrium levels of saving in the economy. In the argument of this example, note from Eq. (5) that the possibility that $\mathrm{d}u \neq 0$ requires precisely the two features described above: that $\mathrm{d}p \neq 0$ and that $\mathrm{E}[\lambda_s(-x_{2,s} + e_2)] \neq 0$.[12]

2. The economy

The economy evolves over two dates, 0 and 1. Individuals are of different types, $i = 1, \dots, I$, and within each type there is a continuum of individuals of mass 1. Individuals of a given type are ex-ante identical, but can face different idiosyncratic shocks at date 1: each individual may find herself in any one of a set of different personal states, $s = 1, \dots, S$. The distribution of individuals of type i across personal states is $\pi^i = (\pi_1^i, \dots, \pi_S^i) \gg 0$: in date 1, a fraction π_s^i of individuals of type i *shall* find themselves in personal state s.

There is a finite number of commodities in the economy, $l = 1, \dots, L$, and individuals consume these commodities in both dates. At date 0, the endowment of an individual depends on her type, and the bundle of commodities is e_0^i. At date 1, individual endowments depend on the type and are subject to idiosyncratic risk: in personal state s, an individual of type i is endowed with

12 When we use specific functional forms to obtain an explicit solution, it is clear from Eq. (6) that relative prices depend on the level of savings, and from Eq. (7) that the weighted aggregate of trades does not vanish, generically (as long as $\varepsilon \neq 0$).

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a bundle e^i_s. Thus, $e^i = (e^i_0, \dots, e^i_s)$ denotes the endowment of an individual;[13] it strictly positive at date 0 and in all personal states at date 1.

While the date 1 endowment of an individual depends on her type and on the idiosyncratic shock, at the macroeconomic level there is no risk:[14] the aggregate endowment of the economy is $\sum_i \sum_{s=1}^S \pi^i_s e^i_s$.

An individual's consumption plan is $x = (x_0, \dots, x_S)$, where each x_s is a bundle of commodities. The preferences of an individual over consumption plans are represented by the utility function[15]

$$u^i(x) = u^i_0(x_0) + \sum_{s=1}^S \pi^i_s u^i_1(x_s),$$

where the temporal, cardinal utility indices, u^i_0 and u^i_1, belong to the class of strictly monotonic, strongly concave, $\mathbf{C}^3$ functions $v : \mathbb{R}^L_{++} \to \mathbb{R}$, that satisfy the following interiority condition: if a sequence $(x_n)_{n=1}^\infty$ of strictly positive consumption bundles converges to a boundary bundle, then $\|Dv(x_n)\|^{-1} Dv\ (x_n) \cdot x_n \to 0$ and $\|Dv(x_n)\|^{-1} \to \infty$. We denote this class of functions by $\mathcal{U}$, and endow it with the topology of $\mathbf{C}^3$, uniform convergence on compacta.[16]

An *economy* is completely described by the profile of endowments and preferences, $(e, u) = ((e^1, u^1), \dots, (e^I, u^I))$.[17] The space of economies is endowed with the product topology.

The economy is sufficiently heterogeneous: (i) individuals are ex-ante heterogeneous, $I \geqslant 2$; (ii) idiosyncratic risk exists, as $S \geqslant 2$; and (iii) the set of commodities is diverse, $L \geqslant 2$.

3. Competitive equilibrium and Pareto efficiency

Allocations of commodities treat all individuals of the same type symmetrically. Thus, an *allocation* for economy (e, u) is a profile $x = (x^1, \dots, x^I)$ that specifies a consumption plan for each type; the allocation is *feasible* if $\sum_i x^i_0 = \sum_i e^i_0$ and $\sum_i \sum_{s=1}^S \pi^i_s x^i_s = \sum_i \sum_{s=1}^S \pi^i_s e^i_s$. The definition of a *Pareto efficient allocation* is the usual one, restricted to the class of type-symmetric allocations.[18]

In order to consider the effects of uninsurable idiosyncratic risk, only a riskless asset can be traded: there is only one financial asset in the economy; it pays one unit of commodity 1 at

[13] For simplicity of notation, state $s = 0$ refers to date 0, whenever there is no possibility of confusion.

[14] All the results in the paper are true in the presence of aggregate risk, even if this risk is fully insurable, as long as idiosyncratic risk remains uninsurable. With aggregate risk, the presentation of the problem and the results is more cumbersome, but the proofs of the theorems are, in fact, simpler.

[15] Again, the arguments would be simpler without additively separability, and if the date-1 Bernoulli indices are state-dependent. On the other hand, the result of the paper continue to hold if the Bernoulli indices of each type of individuals are the same in the two periods, but in this case the proof becomes slightly more complicated.

[16] See [1, §3.17].

[17] The distributions of individuals of a given type across personal states are fixed, and it is not necessary to include these parameters in the definition of an economy.

[18] The usual intuition that at Pareto efficient allocations individual utility functions must have collinear gradients carries over to the present context, but applies in a strong sense: at efficient allocations idiosyncratic risk is fully shared, so (i) for a given type, consumption must be invariant across personal states; and (ii) across types, gradients must be collinear, even for different personal states, given an aggregate state. Formally it suffices to observe that if allocation x is Pareto efficient, then there are strictly positive numbers γ^i, one for each type, such that $\gamma^i Du^i_0(x^i_0) = \gamma^{i'} Du^{i'}_0(x^{i'}_0)$ and $\gamma^i Du^i_1(x^i_s) = \gamma^{i'} Du^{i'}_1(x^{i'}_{s'})$, for all pairs of types, i and i', and all pairs of personal states, s and s'.

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date 1. There is no trade in other assets, in particular in assets that insure against idiosyncratic risk. Holdings of the asset are y, while its price is q.

Besides assets, individuals trade commodities in spot markets. Prices of commodities are $p_0 = (1, \dots, p_{0,l}, \dots)$ at date 0, and $p_1 = (1, \dots, p_{1,l}, \dots)$ at date 1;[19] across dates, prices of commodities are $p = (p_0, p_1)$. Commodity 1 is the numèraire of the economy at each spot, and its price is 1. All other prices are strictly positive, and $\mathcal{P}$ is the set of normalized spot prices, so that, across date events, $p \in \mathcal{P}^2$.

At prices p and q, an individual chooses a consumption plan that maximizes her ex-ante utility, and holdings of the asset that make her consumption plan financially feasible. That is, an individual of type i will choose a plan x and holdings y subject to the constraints that: (i) at date zero, she must be able to afford her portfolio along with current consumption: $p_0(e_0^i - x_0) = qy$; and (ii) in each personal state at date 1, the return of the portfolio, which is simply her asset holdings, must cover the value of her planned consumption, beyond her endowments: $p_1(x_s - e_s^i) = y$, for each personal state s.[20]

The $(S+1) \times L(S+1)$ matrix

$$\Psi(p) := \begin{pmatrix} p_0 & 0 & \dots & 0 \\ 0 & p_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_1 \end{pmatrix}$$

and the $(S+1) \times 1$ vector $R(q) := (-q, 1, \dots, 1)^{\mathsf{T}}$ simplify the statement of the optimization problem of an individual. It is simply

$$\max_{x,y} u^i(x) : \Psi(p)\bigl(x - e^i\bigr) = R(q)y,$$

and the first-order necessary and sufficient conditions are that for some strictly positive vector $\lambda^i = (\lambda_0^i, \dots, \lambda_S^i)$, we have that $Du^i(x^i) = \lambda^i \Psi(p)$ and $\lambda^i R(q) = 0$, while $\Psi(p)(x^i - e^i) = R(q)y^i$.

For an economy (e, u), a *competitive equilibrium* consists of an allocation of commodities, one of asset holdings, a vector commodity prices and a price for the riskless asset, such that all the types of individuals optimize and all markets clear. For a more operational characterization of equilibria, it is convenient for us to make the vectors of individual marginal utilities of income

[19] For simplicity of notation, price vectors and gradients of utility functions are taken as row vectors, whereas quantities are taken as column vectors.

[20] The unique optimizer for individuals of type i, (x^i, y^i) is characterized by the following first-order conditions: for a vector $\lambda^i = (\lambda_0^i, \dots, \lambda_S^i) \gg 0$,

(i) $Du_0^i(x_0^i) = \lambda_0^i p_0$, and $\pi_s^i Du_1^i(x_s^i) = \lambda_s^i p_1$ for all $s = 1, \dots, S$;
(ii) $\lambda_0^i q = \sum_{s=1}^S \lambda_s^i$; and
(iii) $p_0(e_0^i - x_0^i) = qy^i$, and $p_1(x_s^i - e_s^i) = y^i$ at all $s = 1, \dots, S$.

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part of the definition of equilibrium, and to let the first-order conditions of the optimization problems account for individual rationality. Formally, if[21]

$$\mathcal{F}(x, \lambda, y, p, q, e, u) := \begin{pmatrix} \vdots \\ (Du^i(x^i) - \lambda^i \Psi(p))^\top \\ R(q)y^i - \Psi(p)(x^i - e^i) \\ (\lambda^i R(q))^\top \\ \vdots \\ \sum_i (\tilde{e}_0^i - \tilde{x}_0^i) \\ \sum_{i,s} \pi_s^i (\tilde{e}_s^i - \tilde{x}_s^i) \\ \sum_i y^i \end{pmatrix},$$

where, here and henceforth, $\tilde{e}$ and $\tilde{x}$ exclude the numèraire commodity at all states, an equilibrium for the economy (e, u) is a 5-tuple (x, λ, y, p, q) such that $\mathcal{F}(x, \lambda, y, p, q, e) = 0$.

A competitive equilibrium exists for any economy. Importantly, given any profile of preferences, there are only finitely many competitive equilibria, generically in endowments.

Theorem 1. *For any profile of preferences, the set of profiles of endowments for which the economy has finitely many equilibria is open and has full Lebesgue measure.*

Proof. This argument is well known. The result follows from the fact that, given u, $\mathcal{F}(\cdot, u)$ is transverse to 0, and from the Transversality Theorem (see, for example, [12] or [18]). □

A subset of a finite-dimensional Euclidean space is *strongly generic* if it is open and has full Lebesgue measure (its complement has null measure), and a subset of an abstract metric space is *generic* if it is open and dense.

It follows that the model under consideration preserves the standard positive properties of the GEI model with numèraire assets: equilibrium always exists, and there are only finitely many equilibria in a strongly generic set of endowments, for any given profile of preferences. Since idiosyncratic shocks are nontrivial (and uninsurable), it is reasonable to expect that competitive equilibria be inefficient. Indeed, also generically, the inefficiency of competitive equilibrium holds in a strong sense: since idiosyncratic risk is nontrivial, generically in endowments equilibrium consumption depends on all idiosyncratic shocks, for all types of individuals, and is therefore Pareto inefficient.

Theorem 2. *For any profile of preferences, there exists a strongly generic subset of profiles of endowments for which at every competitive equilibrium allocation, x, one has that $\lambda_s^i/\pi_s^i \neq \lambda_{s'}^i/\pi_{s'}^i$ and $x_s^i \neq x_{s'}^i$, for all types i and all personal states $s, s' = 1, \dots, S$, $s \neq s'$.*

---

[21] The domain of $\mathcal{F}$ is

$$\mathbb{R}_{++}^{I(S+1)L} \times \mathbb{R}_{++}^{I(S+1)} \times \mathbb{R}^I \times \mathcal{P}^2 \times \mathbb{R} \times \mathbb{R}_{++}^{I(S+1)L} \times \mathcal{U}^{2I};$$

it maps into

$$\left(\mathbb{R}_{++}^{(S+1)L} \times \mathbb{R}_{++}^{S+1} \times \mathbb{R}\right)^I \times \mathbb{R}_{++}^{2(L-1)} \times \mathbb{R}.$$

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Proof. For each type i, and each pair of date-1 personal states, $s \neq s'$, the mapping

$$(x, y, \lambda, p, q, e) \mapsto \begin{pmatrix} \mathcal{F}(p, q, x, y, \lambda, e, u) \\ \frac{\lambda_s^i}{\pi_s^i} - \frac{\lambda_{s'}^i}{\pi_{s'}^i} \end{pmatrix},$$

by direct computation, is transverse to 0; thus, on a strongly generic set of endowments, the mapping is transverse to 0 as a function of (x, y, λ, p, q) only. Since (x, y, λ, p, q) contains one fewer argument than the mapping has components, this implies that $\lambda_s^i/\pi_s^i \neq \lambda_{s'}^i/\pi_{s'}^i$ whenever $\mathcal{F}(x, y, \lambda, p, q, e, u) = 0$. By the first-order conditions of individual optimization, it follows that $x_s^i \neq x_{s'}^i$ at every equilibrium allocation for endowments in this strongly generic set. The intersection the sets constructed in this way for all (i, s, s') is strongly generic. □

4. Constrained inefficiency

The fact that an allocation is Pareto inefficient says that a reallocation of consumption plans that is feasible from the point of view of the aggregate resources available to the society can improve ex-ante wellbeing for all types. This does not imply, however, that one such reallocation exists which can be implemented through the existing financial instruments.

An allocation x is **constrained-inefficient** if there exist commodity prices $\hat{p}$, a commodity allocation $\hat{x}$, date-zero revenue transfers $(\hat{\tau}^1, \ldots, \hat{\tau}^I)$ and an asset allocation $(\hat{y}^i, \ldots, \hat{y}^I)$ such that

1. revenue transfers are balanced: $\sum_i \hat{\tau}^i = 0$;
2. the asset allocation is feasible: $\sum_i \hat{y}^i = 0$;
3. individual consumptions are optimal (given prices and wealth): for every i, $\hat{x}_0^i$ solves the problem

$$\max_{x'} u_0^i(x') : \hat{p}_0 x' \leqslant \hat{p}_0 e_0^i + \hat{\tau}^i,$$

and each $\hat{x}_s^i$ solves

$$\max_{x'} u_1^i(x') : \hat{p}_1 x' \leqslant \hat{p}_1 e_s^i + \hat{y}^i;$$

4. all markets clear: $\sum_i (e_0^i - \hat{x}_0^i) = 0$ and $\sum_i \sum_{s=1}^S \pi_s^i (e_s^i - \hat{x}_s^i) = 0$; and
5. every individual is ex-ante better off at $\hat{x}$: for every i, $u^i(\hat{x}^i) > u^i(x^i)$.

This is, an allocation is constrained inefficient if a reallocation of wealth, via revenue at date zero and the riskless asset at date one, and competitive trade in the commodity markets can make all types of individual ex-ante better off (condition 5). Conditions 1 and 2 imply that the reallocation is balanced, condition 3 implies that all individuals are rational in the commodity markets, which clear by condition 4.

The main result of this paper is that, typically, equilibrium allocations are constrained inefficient. The theorem has the important implication that just by trading the risk-free asset differently, all types of individuals in the society could be made ex-ante strictly better off.

Theorem 3. *There exists a generic subset of economies, $\mathcal{D}$, where every equilibrium allocation is constrained inefficient: for every $(e, u) \in \mathcal{D}$, if (x, λ, y, p, q) is an equilibrium for (e, u), then x is constrained inefficient.*

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The proof of the theorem exploits the idea of [10]: by perturbing the Hessians of the utility functions, in a local, finite-dimensional subspace of economies, we can change the shape of the demand functions, without changing their level at given prices and endowments (which is makes the set of equilibria invariant in the subspace). These perturbations are used to imply that, generically, relative prices can induce income reallocations beyond the span of the existing asset. For the argument to hold, sufficiently many relative prices (hence commodities) are needed. The argument, which invokes the Transversality Theorem once more, is applied on the finite-dimensional subspace, locally, to obtain constrained inefficiency in a strongly generic subset of that subspace; for the global space of economies, the latter local result implies denseness and hence genericity.

We apply this idea using the technique developed by [6],[22] which allows us to relax a requirement that [10] imposed on the number of commodities: indeed, as in [6], we only require the existence of two commodities.

5. Genericity of constrained inefficiency

A series of properties hold generically at equilibrium and are useful in the proof of Theorem 3. With these properties, for any economy in a generic set, there is associated a lower-dimensional neighborhood of economies. In order to prove that the set of economies where all equilibrium allocations are constrained inefficient is dense, it suffices to show that, for any economy in the generic set, there is an economy in the lower-dimensional neighborhood, arbitrarily close to it, where the property holds.

5.1. A characterization of constrained suboptimality

The function defined by

$$\mathcal{H}(x,\lambda,p,y,\tau,e,u) := \begin{pmatrix} \vdots \\ u^i(x^i) \\ \vdots \\ (Du^i(x) - \lambda\Psi(p))^{\mathsf{T}} \\ \Psi(p)(x^i - e^i) - (\tau^i, 1, \ldots, 1)^{\mathsf{T}} \\ \vdots \\ \sum_i (\tilde{e}^i_0 - \tilde{x}^i_0) \\ \sum_i \sum_{s=1}^{S} \pi^i_s (\tilde{e}^i_s - \tilde{x}^i_s) \\ \sum_i \tau^i \\ \sum_i y^i \end{pmatrix},$$

plays a role similar to $\mathcal{F}$, in the sense that it will make the definition of constrained inefficiency operational, but three differences deserve mention. First, the price of the asset is not an argument, and the block of components that includes the first-order conditions of consumers does not include no-arbitrage conditions for assets: in the alternative plan that makes an allocation constrained inefficient, asset holdings are not being traded or determined by individual optimization.

[22] See, also, [20].

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Second, the first I components of $\mathcal{H}$ are the types' utility levels, which did not appear in $\mathcal{F}$; these components will be used to determine the welfare effects of asset reallocations. Finally, the previous-to-last component of $\mathcal{H}$ captures whether date-0 revenue transfers are balanced.

Lemma 1. *If* (x, λ, y, p, q) *is an equilibrium for economy* (e, u) *and the matrix*

$$D_{x,\lambda,p,y,\tau}\mathcal{H}\big(x, \lambda, p, y, \big(-qy^i\big)_{i=1}^{I}, e, u\big)$$

has full (*row*) *rank, then allocation* x *is constrained inefficient.*

The proofs of this and all other lemmas are in Appendix A.

5.2. *Finite-dimensional subspaces of economies*

All results so far have been independent of the assumption that the economy is commodities are heterogeneous, which implies that there exist relative prices at date 1. The assumption is, nevertheless, important for what follows.

Lemma 2. *For any profile of preferences, on a strongly generic subset of endowments,*

$$\sum_i \sum_{s=1}^{S} \frac{\lambda_s^i}{\lambda_0^i}\big(\tilde{e}_s^i - \tilde{x}_s^i\big) \neq 0$$

at every competitive equilibrium.

Another auxiliary function is obtained by looking only at the lower block of components of the function $\mathcal{H}$, namely the conditions that define competitive equilibrium in commodity trades only, for a given, feasible, allocation of date-0 revenue and holdings of the riskless asset; it is defined by

$$\mathcal{G}(x, \lambda, p, y, \tau, e, u) := \begin{pmatrix} \vdots \\ (Du^i(x) - \lambda\Psi(p))^{\mathsf{T}} \\ \Psi(p)(x^i - e^i) - (\tau^i, 1, \ldots, 1)^{\mathsf{T}} \\ \vdots \\ \sum_i (\tilde{e}_0^i - \tilde{x}_0^i) \\ \sum_{i,s} \pi_s^i(\tilde{e}_s^i - \tilde{x}_s^i) \\ \sum_i \tau^i \\ \sum_i y^i \end{pmatrix}.$$

The set of economies defined in the following lemma will be the basis of the local analysis later.

Lemma 3. *There exists a generic subset of economies,* $\mathcal{D}_r$*, where there is only a finite number of equilibria and, at every equilibrium,*

1. *the matrix* $D_{x,\lambda,y,p,q}\mathcal{F}(x, \lambda, y, p, q, e, u)$ *has full rank;*
2. *for every* i*,* s *and* s'*,* $s \neq s'$*,* $\lambda_s^i/\pi_s^i \neq \lambda_{s'}^i/\pi_{s'}^i$ *and* $x_s^i \neq x_{s'}^i$*;*
3. $\sum_i \sum_{s=1}^{S} \frac{\lambda_s^i}{\lambda_0^i}(\tilde{e}_s^i - \tilde{x}_s^i) \neq 0$*; and*
4. *the matrix* $D_{x,\lambda,p,y,\tau}\mathcal{G}(x, \lambda, p, y, -(qy^i)_{i=1}^{I}, e, u)$ *has full rank.*

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5.3. Finite-dimensional subspaces of economies

For any given economy that satisfies the properties of Lemma 3, there is a neighborhood of economies where the set of equilibria is the same. These economies have the same endowments, but different preferences; the change in preferences is only in their second derivatives at equilibrium consumptions, which implies the invariance of equilibria. The construction first fixes a set of commodity bundles at which it perturbs the utility functions; then, it introduces the perturbations.

The construction is local, so we start by fixing an economy $(\bar{e}, \bar{u})$ in the set $\mathcal{D}_r$ defined in Lemma 3. Let $\mathcal{E}$ denote the set of competitive equilibria of this economy, and notice that, since this set is finite, we can isolate its points in open balls of sufficiently small radius: in particular, let $\bar{\epsilon} > 0$ be such that the open balls of radius $2\bar{\epsilon}$ around each equilibrium contain no other equilibria.[23]

5.3.1. Step 1: Bundles where preferences are perturbed

Perturbations of the utility functions are restricted to neighborhoods of particular consumption levels. For the construction of the perturbations, X_0^i and X_1^i, are for each type, be the levels of date 0 and date 1 consumption bundles at some equilibrium: formally,

$$X_0^i := \left\{x \in \mathbb{R}^L \;\middle|\; \bar{x}_0^i = x \text{ for some } (\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q}) \in \mathcal{E}\right\}$$

and

$$X_1^i := \left\{x \in \mathbb{R}^L \;\middle|\; \bar{x}_s^i = x \text{ for some } (\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q}) \in \mathcal{E} \text{ and some } s = 1, \ldots, S\right\}.$$

By the properties of economy $(\bar{e}, \bar{u})$, all sets X_0^i and X_1^i are finite, and there are $\epsilon^i > 0$ such that the open balls of radius $2\epsilon^i$ isolate these points from one another.[24]

5.3.2. Step 2: Perturbations of preferences

For each $\epsilon > 0$, $\rho_\epsilon : \mathbb{R}^L \to [0, 1]$ denotes a $\mathbf{C}^\infty$ function such that $\rho_\epsilon(\delta) = 1$ in $B_\epsilon(0)$ and $\rho_\epsilon(\delta) = 0$ outside $B_{2\epsilon}(0)$.[25] Also, $(\Delta_{\bar{x}})_{\bar{x} \in X_0^i}$ is an array of symmetric, $L \times L$ matrices with norm less than $\delta > 0$;[26] this array contains one matrix for each of the equilibrium levels of consumption of individuals of type i at date 0. For δ small enough, the function

$$u_0^i(x) = \bar{u}_0^i(x) + \frac{1}{2} \sum_{\bar{x} \in X_0^i} \rho_{\epsilon^i}(x - \bar{x}) \cdot (x - \bar{x})^{\mathsf{T}} \Delta_{\bar{x}} (x - \bar{x})$$

satisfies all properties of utility functions (that is, it lies in the class $\mathcal{U}$). Of course, the same holds for date 1 preferences: for an array $(\Delta_{\bar{x}})_{\bar{x} \in X_1^i}$, containing a symmetric matrix for each date-1

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23 That is, that for every equilibrium $(\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q}) \in \mathcal{E}$, one has that $B_{2\bar{\epsilon}}(\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q}) \cap \mathcal{E} = \{(\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q})\}$.

24 As before, $B_{2\epsilon^i}(\bar{x}) \cap X_0^i = \{\bar{x}\}$, for each $\bar{x} \in X_0^i$, and similarly for bundles in X_1^i.

25 [12], or [6].

26 Formally, each matrix is a real vector with $L(L+1)/2$ components; their norms are computed by treating them as vectors.

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equilibrium consumption, the associated admissible date 1 utility function is

$$u_1^i(x) = \bar{u}_1^i(x) + \frac{1}{2}\sum_{\bar{x}\in X_1^i} \rho_{\epsilon^i}(x-\bar{x}) \cdot (x-\bar{x})^\top \Delta_{\bar{x}}(x-\bar{x}).$$

As a result, there is, for each individual, a new ex-ante utility function $u^i(x) = u_0^i(x_0) + \sum_{s=1}^S \pi_s^i u_1^i(x_s)$, which varies smoothly simply with the arrays of matrices Δ.

More formally, $\bar{\mathcal{U}}_\delta$ is the set of all profiles of ex-ante utility functions that can be obtained by perturbing the corresponding $\bar{u}_0^i$ and $\bar{u}_1^i$ functions as described; this set is a finite-dimensional submanifold of $\mathcal{U}^{2I}$, parameterized by

$$\mathbb{B}_\delta := \prod_i \big(B_\delta(0)^{\#X_0^i} \times B_\delta(0)^{\#X_1^i}\big),$$

with each open ball taken in $\mathbb{R}^{L(L+1)/2}$. Importantly, if one restricts $\mathcal{F}$ to profiles of preferences defined in $\bar{\mathcal{U}}_\delta$ and adopts this parameterization, then $\mathcal{F}$ is twice continuously differentiable. Also, for profiles in this set,

1. for any bundle x, there exists at most one perturbation that is "active" in the sense that there is at most one $\bar{x} \in X_0^i$ such that

$$u_0^i(x) = \bar{u}_0^i(x) + \frac{1}{2} r \cdot (x-\bar{x})^\top \Delta_{\bar{x}}(x-\bar{x}),$$

   for $r > 0$, and similarly, for u_1^i; and
2. at every the equilibrium of the economy $(\bar{e}, \bar{u})$ the perturbations affect only the Hessian of the utility functions, and these are perturbed exactly by the corresponding matrix: for each $(\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q}) \in \mathcal{E}$,

$$u_0^i(\bar{x}_0^i) = \bar{u}_0^i(\bar{x}_0^i), \qquad Du_0^i(\bar{x}_0^i) = D\bar{u}_0^i(\bar{x}_0^i) \quad \text{and} \quad D^2u_0^i(\bar{x}_0^i) = D^2\bar{u}_0^i(\bar{x}_0^i) + \Delta_{\bar{x}_0^i},$$

   and

$$u_1^i(\bar{x}_s^i) = \bar{u}_1^i(\bar{x}_s^i), \qquad Du_1^i(\bar{x}_s^i) = D\bar{u}_1^i(\bar{x}_s^i) \quad \text{and} \quad D^2u_1^i(\bar{x}_s^i) = D^2\bar{u}_1^i(\bar{x}_s^i) + \Delta_{\bar{x}_s^i}$$

   for every date-1 state s.

5.3.3. Invariance of equilibria

By the Implicit Function Theorem, equilibria are locally invariant.

Lemma 4. *There exist $\tilde{\delta} > 0$ and $\tilde{\epsilon} > 0$ such that, for every profile of preferences u in the set $\bar{\mathcal{U}}_{\tilde{\delta}}$ and for every equilibrium $(\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q})$ in $\mathcal{E}$,*

$$\mathcal{F}(x,\lambda,y,p,q,\bar{e},u) = 0 \quad \text{and} \quad (x,\lambda,y,p,q) \in B_{\tilde{\epsilon}}(\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q})$$

for $(x,\lambda,y,p,q) = (\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q})$ and only there.

The importance of this construction is the invariance of equilibria with respect to the perturbations of preferences.

Proposition 1. *There exists $\delta > 0$ such that for every profile of preferences u in $\bar{\mathcal{U}}_\delta$, the set of competitive equilibria of economy $(\bar{e}, u)$ is exactly $\mathcal{E}$.*

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Proof. Let $\epsilon := \min\{\bar{\epsilon}, \tilde{\epsilon}\} > 0$. By continuity of $\mathcal{F}$, there exists $\bar{\delta} > 0$ such that, for every $\Delta \in B_{\bar{\delta}}(0)$, $\mathcal{F}(x, \lambda, y, p, q, \bar{e}, \Delta) = 0$ implies that

$$(x, \lambda, y, p, q) \in \bigcup_{(\bar{x},\bar{\lambda},\bar{y},\bar{p},\bar{q})\in\mathcal{E}} B_\epsilon(\bar{x}, \bar{\lambda}, \bar{y}, \bar{p}, \bar{q}).$$

If $\delta := \min\{\bar{\delta}, \tilde{\delta}\}$, and if $\Delta \in \mathbb{B}_\delta$, then, by Lemma 4, the set of equilibria of economy $(\bar{e}, \Delta)$ is a subset of $\mathcal{E}$. The other inclusion is immediate. □

5.3.4. The local subspace of economies

Given any economy in $\mathcal{D}_r$, the argument for denseness of the set of economies where all equilibrium allocations are constrained inefficient will look for economies where this property holds in the following "neighborhood" of the given economy.

Lemma 5. *Given an economy, $(\bar{e}, \bar{u})$, and a profile of utility perturbations $\Delta \in \mathbb{B}_\delta$, there is a system of equations*

1. *for every i and every $s = 1, \ldots, S$,*

$$\frac{1}{\lambda_0^i}\pi_s^i D\bar{u}_1^i(x_s^i) + \pi_s^i(D^2\bar{u}_1^i(x_s^i) + \Delta_s^i)\beta_s^i - \gamma_s^i p_1^\top + \pi_s^i\tilde{\mathbb{I}}^\top\mu = 0;$$

2. *for every i and every $s = 1, \ldots, S$, $p_1 \cdot \beta_s^i = 0$;*
3. $\sum_i \sum_{s=1}^S \lambda_s^i \tilde{\mathbb{I}}\beta_s^i + \sum_i \sum_{s=1}^S \gamma_s^i(\tilde{\tilde{e}}_s^i - \tilde{\tilde{x}}_s^i) = 0$*; and*
4. *for every i, $\sum_{s=1}^S \gamma_s^i + \eta = 0$,*

where $\mu \in \mathbb{R}$, $\eta \in \mathbb{R}$, $\beta_s^i \in \mathbb{R}^L$ and $\gamma_s^i \in \mathbb{R}$, such that, if $(\bar{e}, \bar{u}) \in \mathcal{D}_r$ and δ is chosen as in Proposition 1*, then there exists a subset of $\bar{\mathcal{U}}_\delta$ of preferences that is*

1. *strongly generic (as subset of $\bar{\mathcal{U}}_\delta$), and*
2. *such that if (x, λ, y, p, q) is a competitive equilibrium of economy $(\bar{e}, \Delta)$, then the system above has no solution.*[27]

5.4. The proof of Theorem 3

The claim is that the set of economies where all equilibrium allocations are constrained inefficient is dense. Since $\mathcal{D}_r$ is generic, it suffices to show that for each $(\bar{e}, \bar{u}) \in \mathcal{D}_r$ there is an economy (e, u), arbitrarily close to $(\bar{e}, \bar{u})$, where the property holds. For this, the endowments is fixed, $e = \bar{e}$, and alternative preferences are in the lower-dimensional "neighborhood" of $\bar{u}$ that is defined by Lemma 5. This neighborhood is $\mathcal{N}$. Since $\mathcal{N}$ has full measure, as a subset of the local subspace of economies constructed around $(\bar{e}, \bar{u})$, if, generically in $\mathcal{N}$, all equilibrium allocations of $(\bar{e}, \Delta)$ are constrained inefficient, then the theorem follows.

[27] The profile of matrices Δ identifies the profile of ex-ante preferences constructed as in Section 5.3.2 above, given $(\bar{e}, \bar{u}) \in \mathcal{D}_r$.

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Lemma 6. *The function*[28]

$$\mathcal{M}(x,\lambda,y,p,q,\Delta,\theta) := \begin{pmatrix} \mathcal{F}(x,\lambda,y,p,q,\bar{e},\Delta) \\ D_{x,\lambda,p,y,\tau}\mathcal{H}(x,\lambda,p,y,(-qy^i)_{i=1}^I,\bar{e},\Delta)^\mathsf{T}\theta \\ \frac{1}{2}(\theta^\mathsf{T}\theta-1) \end{pmatrix}$$

is transverse to 0.

By the Transversality Theorem, Lemma 6 implies that for a generic subset of $\mathcal{N}$, one has that $\mathcal{M}(\cdot,\Delta) \pitchfork 0$. Now, the matrix $D_{x,\lambda,y,p,q,\theta}\mathcal{M}$ has

$$I(S+1)L + I(S+1) + I + 2(L-1) + 1 + I(S+1)L + I(S+1) + 2(L-1) + 2I + 1$$

rows, and only

$$I(S+1)L + (S+1)I + I + 2(L-1) + 1 + I + I(S+1)L + I(S+1) \\ + 2(L-1) + 1 + 1$$

columns, and, since $I \geqslant 2$, it follows that it cannot have full row rank. Then, it must be that on a strongly generic subset of $\mathcal{N}$, the function $\mathcal{M}(\cdot,\Delta)$ is never zero, which means that, in that set, whenever $\mathcal{F}(x,\lambda,y,p,q,\bar{e},\Delta)=0$, it is also true that the matrix

$$D_{x,\lambda,p,y,\tau}\mathcal{H}\big(x,\lambda,p,y,\big(qy^i\big)_{i=1}^I,\bar{e},\Delta\big)$$

has full row rank. It follows then from Lemma 1 that x is constrained inefficient, whenever $\mathcal{F}(x,\lambda,y,p,q,\bar{e},\Delta)=0$.

6. Further examples

The mechanism by which a reallocation of assets can induce a Pareto improvement in Theorem 3 relies precisely on the two effects illustrated in the examples of Section 1. Recall that the requirements for effectiveness of a policy intervention in those examples were the ability of the policy to affect date-1 commodity relative prices, and that the aggregate of date-1 commodity trades weighted by probabilities and marginal utilities of income did not vanish. In the proof of Theorem 3, these two features remain critical. To see this, note the third and fourth properties guaranteed to hold, generically on endowments, by Lemma 3. The third condition is precisely the second feature illustrated by the examples, and allows us to prove Lemma 5, and, hence, Lemma 6.[29] The fourth condition allows the policy intervention to affect relative prices; it is used in the proof of Lemma 6 directly.[30]

The two further examples given next illustrate how the general result extends to economies with richer structures. The first example is similar to the case presented in Section 1.2, but introduces a more general production technology. The second example extends the argument to an

[28] It maps

$$\mathbb{R}_{++}^{I(S+1)L} \times \mathbb{R}_{++}^{(S+1)I} \times \mathbb{R}^I \times \mathcal{P}^2 \times \mathbb{R} \times \mathcal{N} \times \mathbb{R}^{I+I(S+1)L+I(S+1)+2(L-1)+1+1}$$

into

$$\mathbb{R}^{I(S+1)L} \times \mathbb{R}^{I(S+1)} \times \mathbb{R}^I \times \mathbb{R}^{2(L-1)} \times \mathbb{R} \times \mathbb{R}^{I(S+1)L} \times \mathbb{R}^{I(S+1)} \times \mathbb{R}^{2(L-1)} \times \mathbb{R}^{2I} \times \mathbb{R}.$$

[29] See the proof of Claim 2 in Appendix A.

[30] See the proof of Claim 1 in Appendix A.

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economy with overlapping generations. While we leave the general argument for these structures as a subject for future research, it is important to note that the mechanism why a policy can affect ex-ante utility levels in these two cases is exactly the same as in the general result and the two examples above, which we remark at the end of this section.

6.1. A production economy

As in the example presented in Section 1.2, a two-period economy is populated by a continuum of mass 1 of identical individuals. Again, a single commodity is available in the first period, and it can be either consumed or invested: the amount of this commodity that is saved by the individuals becomes their endowment for the second period; we denote this amount by k. In the second period, this level k of capital is combined with a second production factor, labor, to produce a consumption good; denote by l and c, respectively, the amounts of labor used and of consumption good produced at date 1.

In the first period, individuals only consume and save. In the second period, they are endowed with $\bar{l}$ units of labor that they supply inelastically, together with their savings k, in exchange for the consumption good. Initially, there is no risk, so that all the individuals are endowed with $\bar{l}$ at date 1. The over-all utility of the individuals is

$$u(k,c) = -k + v(c),$$

where v is the person's utility index for consumption in the second period. If the consumption good is numèraire of the second period, and if the price of capital as $1+r$ and the price of labor as w, then the date-1 budget of the individuals is

$$\tau(k) = (1+r)k + w\bar{l},$$

which they use to buy consumption, so $c = \tau(k)$.

The technology of production is

$$y = f(k,l) + (1-\delta)k,$$

where function f is assumed to exhibit constant returns to scale. It is immediate, then, from the maximization of profits, that $(1+r) = f_k + (1-\delta)$ and $w = f_l$, while, as response to a perturbation,[31] $\mathrm{d}r = f_{kk}\,\mathrm{d}k$ and $\mathrm{d}w = f_{lk}\,\mathrm{d}k = f_{kl}\,\mathrm{d}k$.

From the optimization of the individuals, it must be that $\lambda(1+r) = 1$, where λ represents the marginal utility of consumption at date 1. This implies that[32]

$$\mathrm{d}u = -\mathrm{d}k + \lambda\big(k\,\mathrm{d}r + \bar{l}\,\mathrm{d}w + (1+r)\,\mathrm{d}k\big) = \frac{k\,\mathrm{d}r + \bar{l}\,\mathrm{d}w}{1+r} = \frac{kf_{kk} + \bar{l}f_{kl}}{1+r}\,\mathrm{d}k,$$

which, since f_k is homogeneous of degree 0, is $\mathrm{d}u = 0$. This equality that confirms that, under certainty, the privately determined level of investment cannot be improved upon.

[31] In these expressions and henceforth, it is simpler to omit the arguments, $(k,\bar{l})$, of function f.

[32] In this case, since there is only one commodity at date 1, the computation of λ is immediate. In the case of multiple commodities, one would express the indirect utility of the individual as $V(\tau) = v(c(p,\tau))$ and use $\lambda = V'$. In general, also, with a consumption plan c, endowment e and prices p, one would have

$$\mathrm{d}u = -\mathrm{d}k + \lambda\big(-c\,\mathrm{d}p + e\,\mathrm{d}p + (1+r)\,\mathrm{d}k\big).$$

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Now, the endowment of labor in the second period is subject to idiosyncratic risk, and is $\bar{l}_s = \bar{l} + \varepsilon_s$ in personal state s, which occurs with probability π_s. As before, $\mathrm{E}[\varepsilon_s] = 0$, and

$$\tau_s(k) = (1+r)k + w\bar{l}_s$$

is the individuals' nominal wealth in personal state s. With ex-ante preferences

$$u(k, c) = -k + \mathrm{E}\big[v_s(c_s)\big],$$

if λ_s is the marginal utility of revenue in state s, the first-order condition of the individuals at date 0 is that $(1+r)\mathrm{E}[\lambda_s] = 1$. As a consequence,

$$\mathrm{d}u = \mathrm{E}\big[\lambda_s(kf_{kk} + \bar{l}_s f_{kl})\big]\,\mathrm{d}k = \mathrm{E}[\lambda_s\varepsilon_s] f_{kl}\,\mathrm{d}k = \mathrm{cov}(\lambda_s, \varepsilon_s) f_{kl}\,\mathrm{d}k, \tag{8}$$

where $\mathrm{cov}(\lambda_s, \varepsilon_s)$ denotes the covariance of the two random variables. The equilibrium allocation is thus constrained suboptimal, as long as $\mathrm{cov}(\lambda_s, \varepsilon_s) \neq 0$. In particular, with state-independent, strictly concave Bernoulli indices, the marginal utility of income is anti-comonotone with the level of consumption, which implies that $\mathrm{cov}(\lambda_s, \varepsilon_s) < 0$, provided that idiosyncratic risk is non-degenerate; in this case, since $f_{kl} > 0$, the expression above implies that individuals *overinvest* at date 0: if $\mathrm{d}k < 0$, then $\mathrm{d}u > 0$.[33]

Once again, for a closed-form solution, the Bernoulli index is

$$v_s(c_s) = \frac{\beta}{\gamma} c_s^{\gamma},$$

where $\beta > 0$ and $\gamma < 1$. In this case, the marginal utilities of income are given by

$$\lambda_s = \beta c_s^{\gamma-1} = \beta(y + \varepsilon_s f_l)^{\gamma-1},$$

so the first-order condition of the individuals' optimization problem is

$$\frac{1}{1+r} = \beta\mathrm{E}\big[(y + \varepsilon_s f_l)^{\gamma-1}\big]$$

while

$$\mathrm{d}u = \beta\mathrm{E}\big[(y + \varepsilon_s f_l)^{\gamma-1}\varepsilon_s\big] f_{kl}\,\mathrm{d}k.$$

If further, there are two equally probable personal states, with $\varepsilon_s = \pm\varepsilon$, then the first order condition becomes

$$\frac{1}{2}\beta(y + \varepsilon f_l)^{\gamma-1} + \frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1} = \frac{1}{1+r}$$

which implies, since $\varepsilon > 0$ and $\gamma < 1$, that

$$\frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1} > \frac{1}{1+r}.$$

Since in this case, by substitution,

$$\mathrm{E}[\lambda_s\varepsilon_s] = \frac{1}{2}\beta(y + \varepsilon f_l)^{\gamma-1}(\varepsilon) + \frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1}(-\varepsilon) = \beta\varepsilon\big(1 - (y - \varepsilon f_l)^{\gamma-1}\big) < 0, \tag{9}$$

which verifies that individuals overinvest at date 0.

[33] For instance, in the case when there only are two equally probable personal states, with $\varepsilon_s = \pm\varepsilon$, by concavity it must be that $\lambda(-\varepsilon) > \lambda(\varepsilon)$, so it follows immediately that $\mathbb{E}_\pi[\lambda_s\varepsilon_s] = \varepsilon(\lambda(\varepsilon) - \lambda(-\varepsilon))/2 < 0$.

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6.2. Overlapping generations

Finally, the economy is one of overlapping generations, as in [9], where each generation lives for two periods, and the population grows at a constant rate $n \geqslant 0$. Suppose, also, that the productive sector is as in the previous example.

Initially, individuals of the economy are endowed with $\bar{l}$ units of labor in *each* of the two periods they live, which they supply inelastically. Using the same notation as in the previous example, their ex-ante utility is given by

$$u(k) = \bar{l}w - k + v\big(\bar{l}w + (1+r)k\big),$$

and the effects of a perturbation are

$$\mathrm{d}u = \bar{l}\,\mathrm{d}w - \mathrm{d}k + \lambda\big(k\,\mathrm{d}r + \bar{l}\,\mathrm{d}w + (1+r)\,\mathrm{d}k\big).$$

In this case, the first-order conditions of individual optimization are that $(1+r)\lambda = 1$, and, by direct substitution,

$$\mathrm{d}u = \frac{(2+r)\bar{l}\,\mathrm{d}w + k\,\mathrm{d}r}{1+r}.$$

On the other hand, since the total supply labor is $(2+n)\bar{l}$, we get, from the fact that the production technology is of constant returns to scale, that

$$(2+n)\bar{l}\,\mathrm{d}w + k\,\mathrm{d}r = 0,$$

so

$$\mathrm{d}u = \frac{r-n}{1+r}\bar{l} f_{lk}\,\mathrm{d}k,$$

which establishes the *Golden Rule* criterion: if the interest rate is above (resp. below) the rate of population growth, in equilibrium the economy underinvests (resp. overinvests).

Alternatively, the endowment of labor in the second period is subject to idiosyncratic risk, and is $\bar{l}_s = \bar{l} + \varepsilon_s$ with probability π_s, where $\mathrm{E}[\varepsilon_s] = 0$. As before, if λ_s be the marginal utility of income in state s, the first-order condition for optimization is that $(1+r)\mathrm{E}[\lambda_s] = 1$, and, hence, the welfare effects of a perturbation are

$$\mathrm{d}u = \bar{l}\,\mathrm{d}w - \mathrm{d}k + \mathrm{E}\big[\lambda_s\big(k\,\mathrm{d}r + \bar{l}_s\,\mathrm{d}w + (1+r)\,\mathrm{d}k\big)\big] = \left(\frac{r-n}{1+r}\bar{l} + \mathrm{cov}(\lambda_s, \varepsilon_s)\right) f_{lk}\,\mathrm{d}k. \quad (10)$$

With state-independent, strictly concave Bernoulli indices, $\mathrm{cov}(\lambda_s, \varepsilon_s) < 0$, and it follows from the latter expression that when the interest rate is below the rate of population growth the competitive equilibrium implies overinvestment (as in the case of certainty). But now, in the presence of idiosyncratic risk, the second prescription of the Golden Rule *may* fail, and in an economy where the interest rate is higher than the growth of population, it may be that a Pareto improvement requires for every generation to save less.

If the Bernoulli indices are state-independent and equal to

$$v(c) = \frac{\beta}{\gamma}c^{\gamma},$$

as in the previous example, and

$$f(k,l) = k^{\alpha} l^{1-\alpha},$$

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Table 1
Welfare effects of a policy perturbation.

n	$\frac{r-n}{1+r}\bar{l}$	$\frac{r-n}{1+r}\bar{l}+\mathrm{cov}(\lambda_s,\varepsilon_s)$
0	0.062004539	0.020834359
0.01	0.050169498	0.009184415
0.02	0.038357781	−0.002443666
0.03	0.026569236	−0.014050019
0.04	0.014803712	−0.02563478
0.05	0.003061059	−0.037198082
0.06	−0.008658872	−0.048740058
0.07	−0.020356227	−0.060260838
0.08	−0.03203115	−0.071760553
0.09	−0.043683786	−0.08323933

then, from the first-order condition, we can solve for the optimal level of investment as

$$k=\alpha^{\frac{1}{1-\alpha\gamma}}\big[(2+n)\bar{l}\big]^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}}\Delta^{\frac{1}{1-\alpha\gamma}},$$

where $\Delta=\beta\mathrm{E}[(\alpha+\delta_s(1-\alpha))^{\gamma-1}]$ and

$$\delta_s=\frac{\bar{l}+\varepsilon_s}{(2+n)\bar{l}}.$$

Also,

$$\lambda_s=\beta c_s^{\gamma-1}=\beta\big(y-(1+n)\bar{l}w+\varepsilon_s w\big)^{\gamma-1},$$

so if we further take that $\varepsilon_s=\pm\varepsilon$ with probability $1/2$, then the first-order condition becomes

$$\mathrm{E}[\lambda_s]=\frac{1}{2}\beta\big(y-(1+n)\bar{l}w+\varepsilon w\big)^{\gamma-1}+\frac{1}{2}\beta\big(y-(1+n)\bar{l}w-\varepsilon w\big)^{\gamma-1}=\frac{1}{1+r},$$

which implies that

$$\frac{1}{2}\beta\big(y-(1+n)\bar{l}w-\varepsilon w\big)^{\gamma-1}>\frac{1}{1+r}.$$

By substitution,

$$\mathrm{E}[\lambda_s\varepsilon_s]=\varepsilon\left(\frac{1}{1+r}-\beta\big(y-(1+n)\bar{l}w-\varepsilon w\big)^{\gamma-1}\right)<0. \tag{11}$$

Moreover, using the equilibrium value of k, in order to determine whether the presence of idiosyncratic risk reverses the classical prescription of the golden rule, it suffices to compute we can compute $\mathrm{E}[\lambda_s\varepsilon_s]$. If the values of the different parameters are $\alpha=\gamma=0.5$, $\bar{l}=1.4$, $\varepsilon=0.6$ and $\beta=0.95$. Table 1 gives the policy prescription, in terms of the sign a Pareto improving perturbation dk, for different values of n.

With these values, if $n\leqslant 0.01$, from the third column of the table, $\mathrm{d}u/\mathrm{d}k>0$, namely that the economy underinvests at equilibrium – and for these values, since $r>n$, the Golden Rule prescribes, similarly, that the economy should invest more. For values of $n\geqslant 0.06$, then $\mathrm{d}u/\mathrm{d}k<0$, so that the Pareto improvement should be induced by a reduction in capital accumulation, which agrees with the recommendation of the Golden Rule, for in these cases $r<n$. But the same is not true for values of $0.02\leqslant n\leqslant 0.05$, where the actual $\mathrm{d}u/\mathrm{d}k<0$ implies that the economy should accumulate *less* capital, while the Golden Rule prescribes the opposite, since $r>n$.

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6.3. The mechanism

The demonstration that these examples illustrate general results is left for future research. Importantly, though, one must note that the mechanism behind the effectiveness of these policy interventions is exactly the same as in the general argument of the paper – and in the two examples given in the introduction.

For the ex-ante homogeneous economy with general production, note from Eq. (8) that a perturbation to the capital stock affects ex-ante utility if, and only if, $f_{kl} \neq 0$ and $\mathrm{cov}(\lambda_s, \varepsilon_s) \neq 0$. The first requirement guarantees that the perturbation has effects on the date-1 wage, the relevant relative price in this one-good economy. The second requirement is the condition that the aggregate, across personal states, of commodity trades weighted by the product of the probability and the marginal utility of revenue does not vanish. In the case of specific functional forms, note from Eq. (9) that effectiveness of the policy requires that $\varepsilon \neq 0$, which is a generic conditions endowments.[34]

The lesson in the case of the OLG economy is slightly different, for in this case the equilibrium allocation may fail Pareto efficiency even in the absence of uncertainty. Our point in this case is that uncertainty introduces a new mechanism by which a policy perturbation can affect ex-ante welfare, and that the standard recommendation derived from the Golden Rule may be invalid. But, again, for this to be the case, two conditions have to be met: from Eq. (10), it requires that $f_{kl} \neq 0$ and $\mathrm{cov}(\lambda_s, \varepsilon_s) \neq 0$. These two requirements have the same interpretation as before, and it follows from the computations for specific functions, in particular from Eq. (11), that the result requires that $\varepsilon \neq 0$.

7. Concluding remarks

The positive properties of the standard general equilibrium model hold in an economy of aggregate and uninsurable idiosyncratic risks. For every profile of endowments and preferences, equilibria exist and, for profiles of preferences in a strongly generic set, the number of equilibria is finite (Theorem 1).

Pareto efficiency requires marginal rates of substitution for commodities to be independent of the individual and of the idiosyncratic shocks; since these shocks are uninsurable, for any profile of preferences, on a strongly generic set of endowments all competitive equilibrium allocations are Pareto inefficient (Theorem 2).

But Pareto efficiency ignores the existence of financial constraints – in particular, the fact that idiosyncratic risks may be uninsurable. A more interesting question is whether the existing assets, which only allow for (perfect) insurance against aggregate shocks, can be used to induce a Pareto improvement over the equilibrium allocations, without requiring that commodity markets be closed. Following the definition of constrained suboptimality [10,19], a simple characterization of equilibria in which that type of social improvement is possible is given by the technique of [6], which we apply here (Lemma 1).

Given a profile of preferences and a profile of endowments where there are finitely many equilibria and all of them are Pareto inefficient (a strongly generic condition), a finite-dimensional space of preferences where the set of equilibria (for the fixed profile of endowments) does not change is constructed (Proposition 1). This space is parameterized by perturbations to the Hes-

[34] It also requires that $\beta \neq 0$, for otherwise the individuals' utilities are constant. This condition is, obviously, generic on preferences.

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sians of the utility functions that do not change their gradients at the equilibrium points (changes to the shape of individual demands that do not change their levels at equilibrium prices). On this finite-dimensional space of preferences, a strongly generic subset has the property that all equilibria are constrained inefficient; this implies that on an open and dense subset of the space of economies, every competitive equilibrium allocation is constrained inefficient (Theorem 3). The result requires that commodities be diverse enough (Lemma 2); this is because the Pareto improvement is generated by the response of relative prices to the perturbation in asset portfolios, which yields transfers of revenue across states of the world which are not available directly from the existing assets: they create insurance opportunities against idiosyncratic risk.

The general argument is motivated by two introductory examples, which illustrate the mechanism by which a financial policy can make all the individuals in the economy ex-ante better off. It requires that the policy be able to perturb future relative prices, and it requires that in the competitive equilibrium individuals fail to perfectly insure their idiosyncratic risk. Loosely speaking, the first requirement is satisfied generically on preferences, while the second one holds generically on endowments. In our general argument, these effects are crucial in the proof of the main result in the paper,[35] and, importantly, they constitute the mechanism by which the ideas of the paper can be extended to economies with general production technologies (even if ex-ante homogeneous) and to economies of overlapping generations, as illustrated by the examples of the paper.

Our argument does not answer the question of what information is necessary for the determination of a Pareto improving financial intervention; it only says that one such intervention typically exists. Existing results on the identification of unobservable fundamentals of the economy, both positive and negative, do not apply to the structure of idiosyncratic risk, so this question remains open. But our argument says that the view that intervention in financial markets cannot induce a Pareto improvement is untenable; it is a different argument to say that productive intervention is impossible because of the inherent difficulty in determining the right policy.

Appendix A. Proofs of the lemmas

Proof of Lemma 1. Since the partial Jacobean has full rank, it follows from the Inverse Function Theorem that $\mathcal{H}(\cdot, e, u)$ maps a neighborhood of $(x, \lambda, p, y, (qy^i)_{i=1}^I)$ onto a neighborhood of $\mathcal{H}(x, \lambda, p, y, (qy^i)_{i=1}^I, e, u)$. It follows that for a small enough $\delta > 0$, there exists $(\hat{x}, \hat{\lambda}, \hat{p}, \hat{y}, \hat{\tau})$ such that

$$\mathcal{H}(\hat{x}, \hat{\lambda}, \hat{p}, \hat{y}, \hat{\tau}, e, u) = \left(u^1\left(x^1\right) + \delta, \dots, u^I\left(x^I\right) + \delta, 0, \dots, 0\right)^{\mathsf{T}},$$

which means that x is constrained inefficient. □

Proof of Lemma 2. Fix a profile of preferences u. Let $\pi := (\pi_1^1, \dots, \pi_S^1, \pi_1^2, \dots, \pi_S^I)^{\mathsf{T}}$, and define the function

$$(x, \lambda, y, p, q, e) \mapsto \begin{pmatrix} \mathcal{F}(x, \lambda, y, p, q, e, u) \\ \sum_i \sum_{s=1}^S \frac{\lambda_s^i}{\lambda_0^i}(\tilde{e}_s^i - \tilde{x}_s^i) \end{pmatrix},$$

for e in the generic subset of Theorem 2.

[35] They are captured by the third and fourth conditions in Lemma 3, which allow us to prove Lemmas 5 and 6; this last lemma is the main argument for Theorem 3.

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Suppose that $(x, \lambda, y, p, q, e) \mapsto 0$. With arguments in the order

$$\left(x^1, \lambda^1, y^1, \ldots, x^I, \lambda^I, y^I, e^1, \ldots, e^I\right),$$

its (partial) Jacobean writes as

$$\begin{pmatrix} D^2u^1(x^1) & -\Psi(p)^\top & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\ -\Psi(p) & 0 & R(q) & \ldots & 0 & 0 & 0 & \Psi(p) & \ldots & 0 \\ 0 & R(q)^\top & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & D^2u^I(x^i) & -\Psi(p)^\top & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & -\Psi(p) & 0 & R(q) & 0 & \ldots & \Psi(p) \\ 0 & 0 & 0 & \ldots & 0 & R(q)^\top & 0 & 0 & \ldots & 0 \\ \Phi^1 & 0 & 0 & \ldots & \Phi^I & 0 & 0 & -\Phi^1 & \ldots & -\Phi^I \\ 0 & 0 & 1 & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 \\ L^1 & z^1 & 0 & \ldots & L^I & z^I & 0 & -L^1 & \ldots & -L^I \end{pmatrix},$$

where, for each i,

$$\Phi^i := \begin{pmatrix} \tilde{\mathbb{I}} & 0 & \ldots & 0 \\ 0 & \pi_1^i \tilde{\mathbb{I}} & \ldots & \pi_S^i \tilde{\mathbb{I}} \end{pmatrix} \quad \text{and} \quad L^i := \begin{pmatrix} \frac{\lambda_1^i}{\lambda_0^i}\tilde{\mathbb{I}} & \ldots & \frac{\lambda_S^i}{\lambda_0^i}\tilde{\mathbb{I}} \end{pmatrix},$$

where $\tilde{\mathbb{I}}$ denotes the L-dimensional identity matrix, with its first row removed. We now argue that this matrix has full row rank, in two steps.

Step 1: By standard arguments, the submatrix without the last superrow and the supercolumns $(e^2, \ldots, e^I)$ has full row rank.

Step 2: When we add last superrow and the $(e^2, \ldots, e^I)$ supercolumns, we add $(L-1)$ rows and $(I-1)(S+1)L > L-1$ columns. Notice that, by Theorem 2, matrix

$$\begin{pmatrix} \pi_1^2 & \pi_S^2 \\ \lambda_1^2 & \lambda_S^2 \end{pmatrix}$$

has full rank. Fix any $l = 2, \ldots, L$. Multiply the columns $e_{l,1}^2$ and $e_{l,S}^2$, respectively, by $\alpha(e_{l,1}^2)$ and $\alpha(e_{l,S}^2)$ such that

$$\begin{pmatrix} \pi_1^2 & \pi_S^2 \\ \lambda_1^2 & \lambda_S^2 \end{pmatrix}\begin{pmatrix} \alpha(e_{l,1}^2) \\ \alpha(e_{l,S}^2) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and multiply the columns $e_{1,1}^2$ and $e_{1,S}^2$, respectively, by $-\alpha(e_{l,1}^2)/p_l$ and $-\alpha(e_{l,S}^2)/p_l$. Leaving all other columns unperturbed, this product gives 1 at the l-th entry of the last superrow and 0 everywhere else. Doing this for all $l = 2, \ldots, L$ gives that the whole matrix has full row rank.

The latter implies that the mapping is transverse to 0 and, hence, that the set of endowments on which it is transverse to 0, as a function of (x, λ, y, p, q) only, has full measure. Since the mapping has $I(S+1)L + I(S+1) + I + 2(L-1) + L$ components and (x, λ, y, p, q) contains only $I(S+1)L + I(S+1) + I + 2(L-1) + 1$ arguments, and since $L \geqslant 2$, it follows that this mapping can be transverse to 0 only if it never takes the value 0. □

Proof of Lemma 3. Theorems 1 and 2 and Lemma 2 give strongly generic sets of endowments where equilibria are finite and satisfy, respectively, the first three properties. For the fourth property, as in Theorem 1, it suffices to observe that $\mathcal{G}(\cdot, u) \pitchfork 0$, and then to invoke the Transversality

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Theorem, to conclude that, for any profile of preferences u, in a strongly generic set of endowments, $\mathcal{G}(\cdot, e, u) \pitchfork 0$. With this result, genericity of the $\mathcal{D}_r$ follows by taking the intersection of these four generic sets. □

Proof of Lemma 4. This follows immediately from the first property of Lemma 3, by the Implicit Function Theorem. □

Proof of Lemma 5. Consider the function

$$\begin{pmatrix} \mathcal{F}(x, \lambda, y, p, q, \bar{e}, \Delta) \\ \vdots \\ \frac{1}{\lambda_0^i}\pi_s^i D\bar{u}_1^i(x_s^i) + \pi_s^i(D^2\bar{u}_1^i(x_s^i) + \Delta_s^i)\beta_s^i - \gamma_s^i p_1^{\mathsf{T}} + \pi_s^i \tilde{\mathbb{I}}^{\mathsf{T}}\mu \\ p_1 \cdot \beta_s^i \\ \vdots \\ \sum_i \sum_{s=1}^S \lambda_s^i \tilde{\mathbb{I}}\beta_s^i + \sum_i \sum_{s=1}^S \gamma_s^i(\tilde{\bar{e}}_s^i - \tilde{\bar{x}}_s^i) \\ \vdots \\ \sum_{s=1}^S \gamma_s^i + \eta \\ \vdots \end{pmatrix}.$$

We are going to show that this matrix is transverse to 0. For this, we first establish that when the function takes value 0, $\beta_s^i \neq 0$ for every i and every $s = 1, \ldots, S$. To see that this is the case, suppose, for instance, that $\beta_1^1 = 0$. Then, it is immediate that

$$\frac{1}{\lambda_0^1}\pi_1^1 D\bar{u}_1^1\big(x_1^1\big) + \pi_1^1 \tilde{\mathbb{I}}^{\mathsf{T}}\mu = 0,$$

and hence, since $\pi_1^1 D\bar{u}_1^1(x_1^1) = \lambda_1^1 p_1$ and $p_{1,1} = 1$, and since the first column of $\tilde{\mathbb{I}}$ is null, we have that $\lambda_1^1/\lambda_0^1 = \gamma_1^1$ and, hence, that $\mu = 0$. Now, the latter implies that for every i and s,

$$\frac{1}{\lambda_0^i}\pi_s^i D\bar{u}_1^i\big(x_s^i\big) + \pi_s^i\big(D^2\bar{u}_1^i\big(x_s^i\big) + \Delta_s^i\big)\beta_s^i - \gamma_s^i p_1^{\mathsf{T}} = 0,$$

so, pre-multiplying by β_s^i, and since, by construction, $\pi_s^i D\bar{u}_1^i(x_s^i) = \lambda_s^i p_1$ and $p_1 \cdot \beta_s^i = 0$, we have that

$$\pi_s^i\big(\beta_s^i\big)^{\mathsf{T}}\big(D^2\bar{u}_1^i\big(x_s^i\big) + \Delta_s^i\big)\beta_s^i = 0,$$

which implies that $\beta_s^i = 0$, and then that $\lambda_s^i/\lambda_0^i = \gamma_s^i$ for all i and s. Now, by condition 3, the latter implies that $\sum_i \sum_{s=1}^S (\lambda_s^i/\lambda_0^i)(\tilde{\bar{e}}_s^i - \tilde{x}_s^i) = 0$, while $\mathcal{F}(x, \lambda, y, p, q, \bar{e}, \Delta) = 0$, which contradicts the fact that $\Delta \in \mathbf{B}_\delta$, by the third condition of Lemma 3.

Now, we need to show that the Jacobean of the function has full row rank when the function takes value of 0. Since, in this case, by construction and Proposition 1, $D_\Delta \mathcal{F}(\cdot) = 0$, it suffices to show that the Jacobean of the rest of the components of the function (i.e., excluding the first component, $\mathcal{F}$) give a full-row-rank Jacobean with respect to $(\beta, \gamma, \mu, \eta, \Delta)$. With the arguments in the order

$$\big(\beta_1^1, \gamma_1^1, \ldots, \beta_S^I, \gamma_S^I, \Delta_1^1, \ldots, \Delta_S^I\big),$$

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the Jacobean writes as

$$\begin{pmatrix} \pi_1^1(D^2\bar{u}_1^1(x_1^1)+\Delta_1^1) & -p_1^\top & \dots & 0 & 0 & N(\beta_1^1) & \dots & 0 \\ -p_1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \pi_S^I(D^2\bar{u}_1^I(x_S^I)\Delta_S^I) & -p_1^\top & 0 & \dots & N(\beta_S^I) \\ 0 & 0 & \dots & -p_1 & 0 & 0 & \dots & 0 \\ \lambda_1^1\tilde{\mathbb{I}} & \tilde{\bar{e}}_1^1-\bar{x}_1^1 & \dots & \lambda_S^I\tilde{\mathbb{I}} & \tilde{\bar{e}}_S^I-\bar{x}_S^I & 0 & \dots & 0 \\ 0 & \mathbf{1}^\top & \dots & 0 & \mathbf{1}^\top & 0 & \dots & 0 \end{pmatrix},$$

where for $t \in \mathbb{R}^L$, $N(t)$ denotes the $L \times L(L+1)/2$ matrix

$$\begin{pmatrix} t_1 & t_2 & t_3 & \dots & t_L & 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & t_1 & t_2 & \dots & 0 & t_2 & t_3 & \dots & t_L & \dots & 0 \\ 0 & 0 & t_1 & \dots & 0 & 0 & t_2 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & t_1 & 0 & 0 & \dots & t_2 & \dots & t_L \end{pmatrix},$$

which has full row rank if (and only if) $t \neq 0$. We argue that the Jacobean has full row rank in a series of steps.

Step 1: The submatrix consisting of the first $2(IS)$ superrows and supercolumns is invertible, by a standard arguments; when we add the other supercolumns and superrows, we add more columns than rows, so it suffices to show that we can perturb the added superrows without perturbing the initial ones, which we do in the following steps.

Step 2: Fix $l = 2, \dots, L$, and define the vector α as follows: $\alpha(\beta_{1,l}^i) = 1/\lambda_1^1$, $\alpha(\beta_{1,1}^1) = -p_{1,l}/\lambda_1^1$, and $\alpha(\chi) = 0$ for every other argument except for Δ_1^1, for which we fix $\alpha(\Delta_1^1)$ such that

$$N\big(\beta_1^1\big)\alpha\big(\Delta_1^1\big) = -\pi_1^1\big(D^2\bar{U}_1^1\big(x_1^1\big) + \Delta_1^1\big)\alpha\big(\beta_1^1\big),$$

which we can do since $N(\beta_1^1)$ contains an invertible $L \times L$ submatrix, given that $\beta_1^1 \neq 0$. Then, the postmultiplication of the Jacobean by α gives 0 in every component, except in the one corresponding to the l-th commodity in the term

$$\sum_i \sum_{s=1}^S \lambda_s^i \tilde{\mathbb{I}} \beta_s^i + \sum_i \sum_{s=1}^S \gamma_s^i \big(\tilde{\bar{e}}_s^i - \tilde{\bar{x}}_s^i\big),$$

where it gives 1.

Step 3: Fix $i = 1, \dots, I$, and define the vector α as follows: $\alpha(\gamma_1^i) = 1$, for all $l = 2, \dots, L$ $\alpha(\beta_{1,l}^i) = -(\bar{e}_{1,l}^i - x_{1,l}^i)/\lambda_l^i$, while $\alpha(\beta_{1,1}^i) = \sum_{l=2}^L p_{1,l}(\bar{e}_{1,l}^i - x_{1,l}^i)/\lambda_l^i$, and $\alpha(\chi) = 0$ for every other argument, except for Δ_1^i, where $\alpha(\Delta_1^i)$ is fixed so that

$$N\big(\beta_1^i\big)\alpha\big(\Delta_1^i\big) = -\pi_1^i\big(D^2\bar{U}_1^i\big(x_1^i\big) + \Delta_1^i\big)\alpha\big(\beta_1^i\big),$$

which we can do since $N(\beta_1^i)$ contains an invertible $L \times L$ submatrix, given that $\beta_1^i \neq 0$. As before, the postmultiplication of the Jacobean by α gives 0 in every component, except in the one corresponding to the term $\sum_{s=1}^S \gamma_s^i + \eta$, where it gives 1.

It follows that the function is transverse to 0, and, hence, that for Δ fixed on subset of $\bar{\mathcal{U}}_\delta$ with full Lebesgue measure (relative to $\bar{\mathcal{U}}_\delta$ itself), the function is transverse to 0 in the rest of

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the arguments. But, as in other arguments, this function has more components than arguments: fixing Δ, it has only

$$I(S+1)L + I(S+1) + I + 2(L-1) + 1 + ISL + IS + (L-1) + 1$$

arguments, and consists of

$$I(S+1)L + I(S+1) + I + 2(L-1) + 1 + ISL + IS + (L-1) + I$$

equations. It follows, then, that the only way in which the function can be transverse to 0, for fixed Δ, is for it to never take the value 0, which proves the result. □

Proof of Lemma 6. Suppose that $\mathcal{M}(x,\lambda,y,p,q,\Delta,\theta)=0$. By Proposition 1, it must be that $\mathcal{F}(x,\lambda,y,p,q,\bar{e},\bar{u})=0$, so, by construction,

$$u_0^i(x_0^i)=\bar{u}_0^i(x_0^i), \qquad Du_0^i(x_0^i)=D\bar{u}_0^i(x_0^i) \quad \text{and} \quad D^2u_0^i(x_0^i)=D^2\bar{u}_0^i(x_0^i)+\Delta_0^i,$$

and

$$u_1^i(x_s^i)=\bar{u}_1^i(x_s^i), \qquad Du_1^i(x_s^i)=D\bar{u}_1^i(x_s^i) \quad \text{and} \quad D^2u_1^i(x_s^i)=D^2\bar{u}_1^i(x_s^i)+\Delta_s^i$$

for every i and s.[36]

Let us name the rows of $D\mathcal{H}$ by[37]

$$(u1,\ldots,uI,f1,b1,\ldots,fI,bI,c0,c1,a,t),$$

so that we can denote θ by

$$\theta^\top := \left(\theta_{u1},\ldots,\theta_{uI},\theta_{f1}^\top,\theta_{b1}^\top,\ldots,\theta_{fI}^\top,\theta_{bI}^\top,\theta_{c0}^\top,\theta_{c1}^\top,\theta_a,\theta_t\right),$$

where $\theta_{ui}\in\mathbb{R}$, $\theta_{fi}\in\mathbb{R}^{(S+1)L}$, $\theta_{bi}\in\mathbb{R}^{S+1}$, $\theta_{c0}\in\mathbb{R}^{L-1}$ $\theta_{c1}\in\mathbb{R}^{L-1}$, $\theta_t\in\mathbb{R}$ and $\theta_a\in\mathbb{R}$. For these vectors, we will further denote by a superindex the state and/or the commodity they correspond to, if applicable (for instance, $\theta_{fi}^s\in\mathbb{R}^L$ and $\theta_{fi}^{s,l}\in\mathbb{R}$).

System $D_{x,\lambda,p,y,\tau}\mathcal{H}(x,\lambda,p,y,(qy^i)_{i=1}^I,\bar{e},\Delta)^\top\theta=0$ can be rewritten as follows:

1. for each i,

$$\theta_{ui}D\bar{u}_0^i(x_0^i)^\top+\left(D^2\bar{u}_0^i(x_0^i)+\Delta_0^i\right)\theta_{fi}^0-\theta_{bi}^0p_0^\top+\tilde{\mathbb{I}}\theta_{c0}=0,$$

   and

$$\theta_{ui}\pi_s^iD\bar{u}_1^i(x_s^i)^\top+\pi_s^i\left(D^2\bar{u}_1^i(x_s^i)+\Delta_s^i\right)\theta_{fi}^s-\theta_{bi}^sp_1^\top+\pi_s^i\tilde{\mathbb{I}}^\top\theta_{c1}=0$$

   for every $s=1,\ldots,S$;
2. for each i, $p_0\theta_{fi}^0=0$ and $p_1\theta_{fi}^s=0$ for every $s=1,\ldots,S$;

---

[36] Here, for simplicity, we are adopting the notation Δ_s^i for $\Delta_{x_s^i}^i$. Also, in what follows we will only consider utility perturbations that are "active" at the given equilibrium, so we take the profile Δ as simply $((\Delta_s^i)_{s=0}^S)_{i=1}^I$.

[37] The logic for this choice is the following: the components of vector θ are identified with the equations of function $\mathcal{H}$; then, ui refers to the utility level of type-i individuals, fi and bi to their first-order and budget-balance conditions, $c0$ and $c1$ to commodity market clearing in both dates, and a and t to the balance required for asset allocations and revenue transfers.

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3. at date 0,

$$\sum_i \lambda_0^i \tilde{\mathbb{I}}\theta_{fi}^0 + \sum_i \theta_{bi}^0 \big(\tilde{\bar{e}}_0^i - \tilde{x}_0^i\big) = 0,$$

while

$$\sum_i \sum_{s=1}^S \lambda_s^i \tilde{\mathbb{I}}\theta_{fi}^s + \sum_i \sum_{s=1}^S \theta_{bi}^s \big(\tilde{\bar{e}}_s^i - \tilde{x}_s^i\big) = 0$$

at date 1; and
4. for every i, $\theta_{bi}^0 + \theta_t = 0$ and $\sum_{s=1}^S \theta_{bi}^s + \theta_a = 0$.

We establish three key properties of this system, by the following claims:

Claim 1. *For at least one type of individuals i, we have that $\theta_{ui} \neq 0$.*

Proof. Suppose, by way of contradiction, that $\theta_{ui} = 0$ for every type. By substituting in condition (1) of the system, this would imply that

$$D_{x,\lambda,p,y,\tau}\mathcal{G}\big(x, \lambda, p, y, \big(qy^i\big)_{i=1}^I, \bar{e}, \Delta\big)^\top \tilde{\theta} = 0,$$

for

$$\tilde{\theta} := \big(\theta_{f1}^\top, \theta_{b1}^\top, \ldots, \theta_{fI}^\top, \theta_{bI}^\top, \theta_{c0}^\top, \theta_{c1}^\top, \theta_a, \theta_t\big)^\top.$$

Since $\mathcal{G}(x, \lambda, p, y, (qy^i)_{i=1}^I, \bar{e}, \Delta) = 0$ and $(\bar{e}, \Delta) \in \mathcal{D}_r$, it follows from the fourth property of Lemma 3 that $\tilde{\theta} = 0$ and hence that $\theta = 0$, which contradicts the fact that $\theta^\top\theta - 1 = 0$. □

Claim 2. *For every type i and date-1 state $s = 1, \ldots, S$, we have that $\theta_{fi}^s \neq 0$.*

Proof. The argument is the same as in the proof that every $\beta_s^i \neq 0$ in Lemma 5, invoking Lemma 2, so details are omitted. □

Claim 3. *For every type i, we have that $\theta_{fi}^0 \neq 0$.*

Proof. As in the proof of Claim 2, if $\theta_{fi}^0 = 0$ for some i, then $\theta_{c0} = 0$ and $\theta_{fj}^0 = 0$ for every $j = 1, \ldots, I$. This implies, by condition (1), that $\theta_{ui}\lambda_0^i = \theta_{bi}^0$, and then, by condition (4), that $\theta_{ui}\lambda_0^i = -\theta_t$ for all i and $s = 1, \ldots, S$. By Claim 1, it must be that $\theta_t \neq 0$, so we can define $\mu = -\theta_t^{-1}\theta_{c1}$, $\eta = -\theta_t^{-1}\theta_a$, $\beta_s^i = -\theta_t^{-1}\theta_{fi}^s$ and $\gamma_s^i = -\theta_t^{-1}\theta_{bi}^s$. By construction, since $\theta_t \neq 0$, vector $(\beta, \gamma, \mu, \eta)$ solves the system defined in Lemma 5, which is impossible. □

Now, to see that $\mathcal{M} \pitchfork 0$, notice that $D\mathcal{M}(x, \lambda, y, p, q, \Delta, \theta)$ writes as

$$\begin{pmatrix} D_{x,\lambda,y,p,q}\mathcal{F}(x, \lambda, y, p, q, \bar{e}, \Delta) & 0 & 0 \\ M & \mathbb{N}(\theta) & D_{x,\lambda,p,y,\tau}\mathcal{H}(x, \lambda, p, y, (qy^i)_{i=1}^I, \bar{e}, \Delta)^\top \\ 0 & 0 & \theta^\top \end{pmatrix},$$

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where

$$\mathbb{N}(\theta) := \begin{pmatrix} N(\theta^0_{f1}) & 0 & \dots & 0 \\ 0 & N(\theta^1_{f1}) & \dots & 0 \\ \vdots & \vdots & \ddots & \dots \\ 0 & 0 & \dots & N(\theta^S_{fI}) \end{pmatrix}$$

for $N(t)$ defined as in the proof of Lemma 5. Since $D_{x,\lambda,y,p,q}\mathcal{F}(x,\lambda,y,p,q,\bar{e},\Delta)$ has full rank, because $(\bar{e},\Delta) \in \mathcal{D}_r$, it suffices that matrix

$$\begin{pmatrix} \mathbb{N}(\theta) & D_{x,\lambda,p,y,\tau}\mathcal{H}(x,\lambda,p,y,(qy^i)^I_{i=1},\bar{e},\Delta)^\top \\ 0 & \theta^\top \end{pmatrix}$$

have full row rank for $D\mathcal{M}(x,\lambda,y,p,q,\Delta,\theta)$ to have full row rank. By Claims 2 and 3, it follows that $\mathbb{N}(\theta^s_{fi})$ has full row rank for all type i and all state, present and future, $s = 0,\dots,S$. An argument similar to the one given in Lemma 5 for transversality of the mapping defined there shows that this matrix has full row rank, and, hence, that matrix $D\mathcal{M}(x,\lambda,y,p,q,\Delta,\theta)$ has full row rank. □

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